Ranging between Mobile Nodes in a Cooperative Positioning Framework

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Ranging between Mobile Nodes in a Cooperative Positioning Framework

Positioning and navigation are of long-standing interest across various fields

Military Applications

Transport Applications

Logistics: Package Tracking

Industry: Object tracking

Science: Geodesy

Sports: Hiking

Healthcare: Patient Monitoring

Search and Rescue

Agriculture: Precision farming

Location based services (LBS)

Mobile Robot Networks: SLAM
Positioning and navigation are of long-standing interest across various fields. Most of location-aware applications assume the availability of real-time updated position information, however, they differ on the level of accuracy and reliability of this information for their proper functionality.
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Most of location-aware applications assume the availability of real-time updated position information, however, they differ on the level of accuracy and reliability of this information for their proper functionality.

- **LBS can work with large errors in position estimates**
  - Routing protocols
  - Data dissemination in LBS e.g. providing information about nearby restaurants to a vehicle
  - etc.

- **Liability critical applications require medium-to-high accuracy**
  - Automatic tolling of road users
  - Speed limit enforcement
  - Tracking of expensive or hazardous material
  - etc.

- **Safety critical and scientific applications require very high accuracy**
  - Autonomous driving
  - ADAS
  - Vehicle active safety system
  - Military Applications
  - Geodetic and scientific applications
  - Precision farming
  - Search and rescue
  - Surveying and mapping
  - Indoor navigation etc.
Positioning and navigation are of long-standing interest across various fields.

Most of location-aware applications assume the availability of real-time updated position information, however, they differ on the level of accuracy and reliability of this information for their proper functionality.

GNSS such as Global Positioning System (GPS) are the most prevalent choice for positioning however, it does not meet the required level of accuracy and availability for most of Liability Critical, Safety Critical and Scientific applications.

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... accuracy and availability ...

Cooperative Positioning (CP) is one of the different paradigms to address this issue where two or more nodes are allowed to work cooperatively for improvement of their individual positions by sharing relevant information.

- Autonomous driving
- ADAS
- Vehicle active safety system
- Military Applications
- Geodetic and scientific applications
- Precision farming
- Search and rescue
- Surveying and mapping
- Indoor navigation etc.
Most popular choice for measurements is the **range** between nodes which is the **distance** measured somehow by the nodes.
Ranging between Mobile Nodes in a Cooperative Positioning Framework

Cooperative Positioning (CP)...

Range measurements between nodes

Localization Techniques

- Sparse vs. Dense
- Anchor-based vs. Anchor-free
- Indoor
- Static vs. Mobile

- Anchor Based
  - Range Based
  - Range Free

- Anchor Free
  - Range Based
  - Range Free
Range-based cooperative positioning is a graph embedding problem where positioning accuracy is affected by three factors:

1. **Range measurement errors**: Gaussian, LoS, NLoS etc.

2. **Geometry of nodes**: Nodes deployed independently and uniformly in a square, sphere etc.

3. **Network parameters (connectivity and size)**: Anchor degree, Sensor degree, vertex connectivity or edge connectivity etc.
Range measurements between nodes

Some common measurement methods are:

- Acoustic energy
- Received signal strength indicator (RSSI)

- Time of arrival (ToA)
- Time difference of arrival (TDoA)
- Round Trip Time (RTT)
- Hybrid Approaches

- Angle of arrival (AoA)
Received signal strength (RSS)

How?
- RSS can be modeled as a linearly decreasing variable on logarithmic scale of the distance. We can consider the RSS of acoustic, RF, or other signals.
- In free space, signal power decays proportional to $\frac{1}{d^2}$ where $d$ is the distance b/w transmitter and receiver.

$$\hat{d}_i = d_o 10^{10n_p} + e_{rss}$$

$$= d_i + e_{rss}$$

$e_{rss} \sim \text{log-normal Random Variable}$
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- Received signal strength (RSS)

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- In free space, signal power decays proportional to $\frac{1}{d^2}$ where $d$ is the distance b/w transmitter and receiver.

\[
\hat{d}_i = d_o 10^{\frac{P_o - P_i}{10n_p}} + e_{rss}
\]

\[
= d_i + e_{rss}
\]

$e_{rss} \sim \log$-normal Random Variable

**Pros:**

- RSS measurements are relatively **inexpensive and simple to implement** in hardware.
- RSS of RF signals can be measured by each receiver during normal data communication **without** presenting additional bandwidth or energy requirements.
- **RSS difference** between two sensors indicates information about their relative distance from the transmitter and removes the dependency on the actual transmit power.
- Are used for **AoA** measurements

**Cons:**

- **Very poor ranging accuracy** as in real-world channels, **multipath signals (frequency selective portion which must be compensated)** and **shadowing (totally random part which must be modeled)** are two major sources of environment dependence in the measured RSS.
- **Path loss exponent must be known** – through campaign; otherwise an additional unknown.
- Highly dependent on **calibration**.
- Only suitable for **dense and short range** networks
• Angle of arrival (AoA)

How?

• By providing information about the direction to neighboring sensors rather than the distance to neighboring sensors, AoA measurements provide localization information complementary to the ToA and RSS measurements.
• AoA can be measured in two ways:
  1) **Array signal processing:** AOA is estimated from the differences in arrival times for a transmitted signal at each of the sensor array elements.
  2) **RSS ratio** between two (or more) directional antennas located on the sensor: Two directional antennas pointed in different directions, such that their main beams overlap, can be used to estimate the AOA from the ratio of their individual RSS values.
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**Pros:**
- **No synchronization** is needed.
- Errors are mostly **Gaussian** and can be modeled adequately well.
- **Good accuracies** are achieved.

**Cons:**
- **Very complex hardware** -> multiple antenna system.
- **Sophisticated signal processing** is required in case of multipath.
- **Calibration** is a challenging task.
- Geometrically, the spatial resolution \( \alpha \) of the intersection of two perfectly complementing AoA measurements is limited to \( 2 d \sin (0.5\alpha) \).
• **Time of Arrival (ToA)**

**How?**
- ToA is the measured time at which a signal (RF, acoustic, or other) first arrives at a receiver. The measured ToA is the time of transmission (ToT) plus a propagation-induced time delay called time of flight (ToF) or signal travel time. The ToA is measured using correlation with locally generated signal replica inside receivers -> **Generalized Cross Correlator (GCC)**
- **When clocks are synchronized**, ToF can be computed by differencing ToT and ToA which in turn is used to compute the range as:
  \[
  \hat{d}_i = c \left( t_{ToA} - t_{ToT} \right) \\
  = d_i + e_{ToA}
  \]
  
  \[e_{ToA} \sim \text{Zero Mean Gaussian}\] if LoS conditions
  
  \[e_{ToA} \sim \text{Positive Bias + Zero Mean Gaussian}\] if NLoS conditions
Ranging between Mobile Nodes in a Cooperative Positioning Framework

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\[e_{ToA} \sim \text{Zero Mean Gaussian} \quad \text{if LoS conditions}\]

\[e_{ToA} \sim \text{Positive Bias + Zero Mean Gaussian} \quad \text{if NLoS conditions}\]

**Pros:**

- Well defined bounds and models for estimation accuracies. We know what to do to achieve theoretical bounds. For a given bandwidth and signal-to-noise ratio (SNR). CRLB is very well-known (in LoS conditions): \( \text{var}(t_{ToA}) \geq \frac{1}{8\pi^2 F_s^2 B T_s SNR} \)
- High accuracy is possible: **cm level accuracy** using UWB signals has also been achieved.
- Suitable for **long distance ranging**.

**Cons:**

- **NLoS is really an issue**: early-arriving paths change peak location while late-arriving path attenuate the signal.
- Wider bandwidths are required for good performance against early-arriving multipath -> **Complex** hardware and a lot of signal processing !
- **Hardware delays** of transmitter and receiver are problematic too -> good calibration required.
- **Synchronization** of clocks is not possible in real scenarios -> clock biases and drifts. For ToA in asynchronous networks, a common practice is to use two-way (or round-trip) TOA measurements or TDoA.
**Ranging between Mobile Nodes in a Cooperative Positioning Framework**

- **Time Difference of Arrival (TDoA)**

**How?**

- In asynchronous networks, ToA measurements become: \( \hat{t}_{\text{ToA}} = t_{\text{ToA}} + \tau_c \) where \( \tau_c \) is clock misalignment error between transmitter and receiver clocks – an additional clock bias term which must be estimated.
- Now, when we form range measurements, we get pseudoranges instead of actual range:

\[
\rho_i = c \left( \hat{t}_{\text{ToA}} - t_{T_{\text{ToT}}} \right) = c \left( t_{\text{ToA}} + \tau_c - t_{T_{\text{ToT}}} \right) = d_i + c\tau + e_{\text{ToA}}
\]

- Now, if all transmitters are synchronized, the unknown bias term will be same for each transmitter equation; hence can be cancelled by taking a difference of ToA measurements -> TDoA

\[
\Delta_{ij} = \rho_i - \rho_j = d_i - d_j + e_{T_{\text{DoA}}} \quad \text{\(e_{T_{\text{DoA}}}: \text{very difficult to model its statistics in the presence of NLoS}\)}
\]

- So, two approaches: 1) either treat unknown clock bias as an additional unknown and estimate it along with node’s position or 2) make TDoA measurements by taking difference of pseudoranges.
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• Time Difference of Arrival (TDoA)

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• In asynchronous networks, ToA measurements become: \( \hat{t}_{ToA} = t_{ToA} + \tau_c \) where \( \tau_c \) is clock misalignment error between transmitter and receiver clocks – an additional clock bias term which must be estimated.

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\]

• So, two approaches: 1) either treat unknown clock bias as an additional unknown and estimate it along with node’s position or 2) make TDoA measurements by taking difference of pseudoranges.

Pros:

• Suitable in more realistic asynchronous scenarios
• High accuracy is possible
• Suitable for **long distance ranging**.

Cons:

• **NLoS is really an issue**: very difficult to model the error terms
• **Synchronization** between known anchor clocks may not be possible in real
• **Differencing** reduces one equation as one node must be selected as reference -> more measurements than ToA and RSS
Other alternatives are also available especially RTT and hybrid approaches.

Here, we focus on some variants of ToA based ranging technique to measure the range between nodes, where nodes are allowed to share their sensed data/measurement from Anchor Nodes, to their neighbors using a wireless link.

I am at a distance $d_1, d_2, d_3, \ldots$ from red, green, yellow, ... anchors respectively. What is your distance from red?

I am at a distance $d_a, d_b, d_c, \ldots$ from red, green, yellow, ... anchors, respectively.
Ranging between Mobile Nodes in a Cooperative Positioning Framework

Other alternatives are also available especially RTT and hybrid approaches.

Direct range measurement

Here, we focus on **some variants of ToA based ranging technique** to measure the range between nodes, where nodes are allowed to share their sensed data/measurement from Anchor Nodes, to their neighbors using a wireless link.

I am at a distance $d_1, d_2, d_3, \ldots$ from red, green, yellow, ... anchors respectively. What is your distance from red?

I am at a distance $d_a, d_b, d_c, \ldots$ from red, green, yellow, ... anchors, respectively.

Ok. Let me compute my distance from you.

Hmmm ... I can also compute my distance from you in same fashion.
Range-based cooperative positioning ... A high-dimensional optimization problem that finds a vector of node locations such that internode distances are as close to range measurements as possible

- \( \rho^i_v(t) \): is the range or pseudorange as measured by node \( v \) from node \( i \) at time \( t \)
- Assume \( i = 1, 2, \ldots, N \) and all Anchor Nodes are synchronized.
- Further, assume, \( v = A, B \)
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- $\rho_v^i(t)$: is the range or pseudorange as measured by node $v$ from node $i$ at time $t$
- Assume $i = 1, 2, \ldots, N$ and all Anchor Nodes are synchronized.
- Further, assume, $v = A, B$

$$\rho_v^i(t) = r_v^i(t) + t_v(t) + \epsilon_c^i(t) + \epsilon_{ud}^i(t)$$
Range-based cooperative positioning … A high-dimensional optimization problem that finds a vector of node locations such that internode distances are as close to range measurements as possible.

\[ \rho^i_v(t) = r^i_v(t) + t_v(t) + \epsilon^i_c(t) + \epsilon^i_{uv}(t) \]

Measured from **cross-correlation** between received signal and local replica.
Range-based cooperative positioning ... A high-dimensional optimization problem that finds a vector of node locations such that internode distances are as close to range measurements as possible.

\[ \rho_{D}^{i}(t) = r_{D}^{i}(t) + t_{D}(t) + \epsilon_{C}^{i}(t) + \epsilon_{U_{D}}^{i}(t) \]

True range

Node clock offset

Correlated error terms

Uncorrelated error terms

\[ r_{v}^{i}(t) = \|p_{i} - p_{v}\| = \sqrt{(x_{i} - x_{u})^{2} + (y_{i} - y_{u})^{2} + (z_{i} - z_{u})^{2}} \] : True range

\[ p_{i} = [x_{i} \quad y_{i} \quad z_{i}]^{T} \] : Anchor node i position vector

\[ p_{v} = [x_{v} \quad y_{v} \quad z_{v}]^{T} \] : Node v position vector
Where is this model applicable? … GNSS systems

All satellite clocks are **synchronized**

Mobile nodes represent any outdoor mobile network e.g. **Vehicular ad-hoc network**

Mobile node clocks are at some **offset** from satellite clocks

**Correlated errors:** Ionosphere + Troposphere + satellite clock and ephemeris errors

**Uncorrelated errors:** noise + multipath + NLoS + clock residual errors
Range-based cooperative positioning ... A high-dimensional optimization problem that finds a vector of node locations such that internode distances are as close to range measurements as possible.

\[
\rho^i_d(t) = r^i_d(t) + t_v(t) + \epsilon^i_c(t) + \epsilon^i_{uv}(t)
\]

\[
y_A = \begin{bmatrix} \rho^i_A \\ \vdots \\ \rho^N_A \end{bmatrix} = A(\theta) + e \quad \text{with} \quad \theta = \begin{bmatrix} p_A \\ t_A \end{bmatrix} = \text{Vector of unknowns}
\]

\(A(\theta)\) is a non-linear mapping; location estimation can be seen as an inverse-problem to find a mapping \(M(.)\) such that \(\hat{\theta} = M(y)\).
Range-based cooperative positioning ... A high-dimensional optimization problem that finds a vector of node locations such that internode distances are as close to range measurements as possible

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-----> **Many algorithms exist to solve it:**

**Non-linear least-square**
- Steepest-descent
- Newton method
  - Trust region
  - Gauss newton
  - Levenberg-Marquardt
  - etc.

**Maximum-likelihood**
Only possible if joint pdf of measurements is known

**Direct Methods**
- Single Objective Function (SOF) based methods
- Multiple Objective Function (MOF) based methods
  - 1D-Iteration (1DI) methods
    - 1DI Type I Methods
    - 1DI Type II Methods
    - etc.

**Bayesian Methods**
- KF, EKF, UKF, PF etc.
Our goal: estimate the distance $\Delta \mathbf{p}(t)$ between nodes A and B using already measurement pseudoranges from Anchor nodes:

$$\rho_v^i(t) = r_v^i(t) + t_v(t) + \epsilon_c^i(t) + \epsilon_{uv}^i(t)$$
Our goal: estimate the distance \( \Delta \mathbf{p}(t) \) between nodes A and B using already measurement pseudoranges from Anchor nodes:

\[
\rho^i_B(t) = r^i_B(t) + t_B(t) + \epsilon^i_c(t) + \epsilon^i_u(t)
\]

\( \rho^i_A(t) \): Position of node A
\( \rho^i_B(t) \): Position of node B
\( \rho^i(t) \): Position of \( i \)-th satellite

\( \mathbf{p}_B(t) = \mathbf{p}_A(t) + \Delta \mathbf{p}(t) \)
**Approach 1:** use raw pseudoranges to estimate $\Delta \mathbf{p}(t)$

\[
\rho^i_v(t) = r^i_v(t) + t_v(t) + \epsilon^i_c(t) + \epsilon^i_{u_v}(t)
\]

Linearize both of these equations around an assumed known point $\mathbf{p}_u(t)$ and then iteratively refine the estimates.
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**Approach 1:** use raw pseudoranges to estimate $\Delta \mathbf{p}(t)$

\[
\rho_A^i(t) = \|\mathbf{p}_i(t) - \mathbf{p}_A(t)\| + t_A(t) + \epsilon_c^i(t) + \epsilon_{uA}^i(t)
\]
\[
\rho_B^i(t) = \|\mathbf{p}_i(t) - \mathbf{p}_A(t) - \Delta \mathbf{p}(t)\| + t_B(t) + \epsilon_c^i(t) + \epsilon_{uB}^i(t)
\]

After linearization and some manipulations, we can formulate a **Least-Square (LS)** problem as:

\[
\Delta \rho_{AB}(t) = \begin{bmatrix}
H_1 & 0_{N \times 4}
\end{bmatrix}
\begin{bmatrix}
\Delta \mathbf{p}_A(t) \\
\Delta t_A(t) \\
\Delta \tilde{\mathbf{p}}_A(t) \\
\Delta \tilde{t}_A(t)
\end{bmatrix} = \begin{bmatrix}
\Delta \tilde{x}_A(t) \\
\Delta \tilde{y}_A(t) \\
\Delta \tilde{z}_A(t)
\end{bmatrix} = \Delta \mathbf{p}_A(t) + \Delta \mathbf{p}(t)
\]

Can be solve iteratively for $\Delta \mathbf{p}(t)$ until the solution is converged
Ranging between Mobile Nodes in a Cooperative Positioning Framework

**Approach 2:** use single difference of raw pseudoranges to estimate $\Delta p(t)$

\[
\rho^i_A(t) = \|p_i(t) - p_A(t)\| + t_A(t) + \epsilon^i_c(t) + \epsilon^i_{uA}(t) \\
\rho^i_B(t) = \|p_i(t) - p_A(t) - \Delta p(t)\| + t_B(t) + \epsilon^i_c(t) + \epsilon^i_{uB}(t)
\]

If the true range between node $A$ and $B$ is not large, then, we can assume that correlated error term is same for both node equations.
**Approach 2:** use single difference of raw pseudoranges to estimate $\Delta \mathbf{p}(t)$

\[
\rho_A^i(t) = \| \mathbf{p}(t) - \mathbf{p}_A(t) \| + t_A(t) + \epsilon_c^i(t) + \epsilon_{uA}^i(t) \\
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\]

If the true range between node $A$ and $B$ is not large, then, we can assume that **correlated error** term is same for both node equations.

\[
S_{AB}^i(t) = \rho_A^i(t) - \rho_B^i(t) \\
= \Delta r_{AB}^i(t) + \Delta t_{AB}(t) + \Delta \epsilon_u^i(t)
\]

New measurement which is the difference of pseudorange of Node A and pseudorange of Node B, measured from same satellite.

How to approximate it?

Although, the contribution due to common error terms is removed in this new observable but contribution of uncorrelated error terms has been increased.
**Approach 2:** use single difference of raw pseudoranges to estimate $\Delta \mathbf{p}(t)$

$$S_{AB}^i(t) = \rho_A^i(t) - \rho_B^i(t)$$

$$= \Delta r_{AB}^i(t) + \Delta t_{AB}(t) + \Delta e_u^i(t)$$

How to approximate it?

Since, true ranges and are much larger than the distance between vehicles, we can approximate the difference in true ranges using inner product between displacement vector and the unit vector pointing towards the direction of $i$-th satellite.
**Approach 2:** use single difference of raw pseudoranges to estimate $\Delta p(t)$

$$S_{AB}^i(t) = \rho_A^i(t) - \rho_B^i(t)$$
$$= \Delta r_{AB}^i(t) + \Delta t_{AB}(t) + \Delta e_u^i(t)$$

$$\begin{bmatrix} S_{AB}^1(t) \\ S_{AB}^2(t) \\ \vdots \\ S_{AB}^N(t) \end{bmatrix} \approx \begin{bmatrix} [e^1]^T & 1 \\ [e^2]^T & 1 \\ \vdots & \vdots \\ [e^N]^T & 1 \end{bmatrix} \begin{bmatrix} \Delta p(t) \\ \Delta t_{AB}(t) \end{bmatrix}$$

Can be solve iteratively for $\Delta p(t)$ until the solution is converged.
Ranging between Mobile Nodes in a Cooperative Positioning Framework

**Approach 3:** use another difference of single differences of raw pseudoranges to estimate $\Delta \mathbf{p}(t)$

$$S^i_{AB}(t) = \rho^i_A(t) - \rho^i_B(t)$$

$$= \Delta r^i_{AB}(t) + \Delta t_{AB}(t) + \Delta \epsilon^i_u(t)$$

In order to further suppress user clock offsets, vehicle A can perform another difference of SD observables corresponding to different satellites $i$ and $j$ to get a new observable.
**Approach 3:** use another difference of single differences of raw pseudoranges to estimate $\Delta \mathbf{p}(t)$

\[
S_{AB}^i(t) = \rho_A^i(t) - \rho_B^i(t) = \Delta r_{AB}^i(t) + \Delta t_{AB}(t) + \Delta \epsilon_u^i(t)
\]

In order to further suppress user clock offsets, vehicle A can perform another difference of SD observables corresponding to different satellites $i$ and $j$ to get a new observable $D_{ij}(t)$

\[
D_{ij}(t) = S_{AB}^i(t) - S_{AB}^j(t) = \Delta R_{ij}(t) + \Sigma_{ij}(t)
\]

New measurement which is the difference of Single Differences for two different satellites

\[
\Delta R_{ij}(t) = [e^i - e^j]^T \Delta \mathbf{p}(t)
\]

Although the contribution due to user clock offsets has been removed but the error contribution due to uncorrelated errors is further increased.
**Approach 3:** use another difference of single differences of raw pseudoranges to estimate $\Delta \textbf{p}(t)$

$$
\begin{bmatrix}
    D^{10}(t) \\
    D^{20}(t) \\
    \vdots \\
    D^{N0}(t)(t)
\end{bmatrix} \approx 
\begin{bmatrix}
    [\text{e}^1 - \text{e}^0]^T \\
    [\text{e}^2 - \text{e}^0]^T \\
    \vdots \\
    [\text{e}^N - \text{e}^0]^T
\end{bmatrix} \Delta \textbf{p}(t)
$$

Can be solved iteratively for $\Delta \textbf{p}(t)$ until the solution is converged.
Some Results

Three scenarios:

- **Static**
- **Highway**
- **Dense Urban**

ANavS Sensor Module (uBlox M8T receiver)

Vehicle equipped with Laptop

Corrections over internet

- Precise RTK position of vehicle with GNSS/INS tight coupling
- Raw data for post-processing

ANavS Reference Station
## Some Results

<table>
<thead>
<tr>
<th>Conditions</th>
<th>True distance between vehicles [m]</th>
<th>Number of records</th>
<th>Duration of each record</th>
</tr>
</thead>
<tbody>
<tr>
<td>Day 1: Experiment 1</td>
<td>3</td>
<td>5</td>
<td>45 min.</td>
</tr>
</tbody>
</table>

**Static**

APD: Absolute Position Difference
PR: Raw Pseudoranges
**SD: Single Difference**
**DD: Double Difference**
Some Results: Effect of uncorrelated errors

\[ \rho^i_D(t) = r^i_D(t) + t^i_v(t) + \epsilon^i_c(t) + \epsilon^i_{uv}(t) \]

- True range
- Node clock offset
- Correlated error terms
- Uncorrelated error terms

\[ \epsilon^i_{uv}(t) = c_v \epsilon^i_{uv}(t - 1) + n_v(t) \]

\[ n_v(t) \sim \mathcal{N}(0, \sigma^2_v) \]
Some Results: Effect of uncorrelated errors

$$\epsilon^i_{uv}(t) = c_v \epsilon^i_{uv}(t-1) + n_v(t)$$

$$n_v(t) \sim \mathcal{N}(0, \sigma^2_v)$$

- APD: Absolute Position Difference
- PR: Raw Pseudoranges
- SD: Single Difference
- DD: Double Difference
Highway

RTK Trajectory
Some Results

Highway

Ranging between Mobile Nodes in a Cooperative Positioning Framework

RTK Trajectory

**APD**: Absolute Position Difference
**PR**: Raw Pseudoranges
**SD**: Single Difference
**DD**: Double Difference
Some Results

Dense Urban

RTK Trajectory
**Some Results**

**Dense Urban**

RTK Trajectory

- **APD**: Absolute Position Difference
- **PR**: Raw Pseudoranges
- **SD**: Single Difference
- **DD**: Double Difference
Conclusions and emerging research directions

- Ranging is a fundamental task for localization and its estimate directly effect the quality and obtainable precision of location information.

- Although they require complex hardware and signal processing, ToA based techniques and their hybrid versions are still preferred choice for ranging between mobile nodes.

- Care must be taken in designing any system/network relying on ToA based ranging as there is a great trade-off among these approaches depending on application.
Conclusions and emerging research directions

- Ranging is a fundamental task ...

<table>
<thead>
<tr>
<th>Energy efficient localization</th>
<th>State-of-art convex optimization</th>
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<tbody>
<tr>
<td>More research works are required to relate powers of anchor nodes to the performance of localization.</td>
<td>A lot of recent works have been published on localization that implement convex optimization and SDP. Mathematical complexity is a challenge.</td>
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<tr>
<th>Anchor free optimization method</th>
<th>Security and privacy</th>
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<tr>
<td>Inaccurate initial guess leads to local minima problems</td>
<td>Malfunctioned sensor, false position, unreliable information exchanges, network attacks ...</td>
</tr>
</tbody>
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New Player in the town: Deep Learning

On Deep Learning-based Massive MIMO Indoor User Localization

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