# Structural Effects and Aggregation in a Social-Network Model of Natural Resource Consumption

**Talha Manzoor**[*,**] **Elena Rovenskaya**[***,****]
**Abubakr Muhammad**[*,**]

[*] *Department of Electrical Engineering, Lahore University of Management Sciences (LUMS), Lahore, Pakistan.*
[**] *Center for Water Informatics & Technology (WIT), Lahore University of Management Sciences (LUMS), Pakistan.*
[***] *Advanced Systems Analysis Program, International Institute for Applied Systems Analysis (IIASA), Laxenburg, Austria.*
[****] *Faculty of Computational Mathematics and Cybernetics, Lomonosov Moscow State University, Moscow, Russia.*

Abstract In this paper we consider a networked system of natural-resource consumption, where the agents are governed by a recently reported model of consumer psychology. Dynamics of the consumption for each agent are influenced by the state of the resource and consumption of her neighbors. The process is parameterized by the psychological characteristics of each agent. This study extends the original model to incorporate the underlying network topology, and explores the effects of aggregation via densely-connected communities present in the network. The exercise yields new interpretations of the predictions made by the original model in context of the influence network. We present aggregation mechanisms first for certain canonical structures of the consuming population and later on for a class of non-canonically structured populations. In the end we present an approximate aggregation scheme for populations where only some inaccurate information of the consumer characteristics is available.

*Keywords:* Natural resources management, Social Networks, Modeling and identification of environmental systems, Control in economics, Multi-agent systems.

## 1. INTRODUCTION

In recent years, the controls community has displayed an increasing interest in human-in-the-loop control systems, which are encountered extensively in the real world as complex socio-technical, socio-economic or socio-ecological systems, to name just a few. Conventional approaches commonly treat human influence in these systems either as external disturbances or exogenous inputs. The human-in-the-loop approach offers a new perspective, whereby humans are treated as fundamental components of the system which may or may not adapt to changes in the surrounding environment. Such a method of inquiry not only results in a more accurate representation of reality, but also uncovers hidden relationships and feedback effects which would otherwise not be taken into account. This presents exciting applications in a diverse range of areas which include (but are not limited to) sociology (Friedkin (2015)), energy management (Nguyen and Aiello (2013)), robotics (Schirner et al. (2013)) and economics (Jager et al. (2000)). In this paper we concern ourselves with the control of socio-ecological systems.

One of the major challenges for human-in-the-loop control is the extension of conventional system-identification techniques in order to accurately model human behavior (Munir et al. (2013)). For socio-ecological systems, various models have been proposed which explicitly account for human behavior (see Anderies (2000); Perman (2003)). However a majority of these models assume that humans act as maximizers of certain objectives (for instance financial profit), and that they are constrained by some common notion of rationality. Human psychology is seldom incorporated in such models, which requires investigating the micro-level interactions that exist between humans and the environment and understanding the factors that drive human behavior in such coupled human-natural systems. There exists a plethora of research conducted by the social-psychology community regarding the various determinants that govern the human behavior in a natural resource crisis (Jager (2000)), commonly referred to as the Tragedy of the Commons (Perman (2003)). While multiple computational (Bousquet and Le Page (2004); Hare and Deadman (2004)) and conceptual (Jager and Mosler (2007)) models of natural resource consumption have been put forward in the past, there remains a dearth of relevant mathematical models that are also compliant with the control-theoretic framework.

Our point of departure is a mathematical model of natural resource consumption recently reported by the authors (Manzoor et al. (2016)). This mathematical model is based on the conceptual Social Ecological Relevance model of Mosler and Brucks (2003) which is derived from the psychological first principles of Festinger's theory of social comparison processes (see Festinger (1954)). The assumed setting of the model is that of an open-access natural resource (Perman (2003)), shared by a consuming population whose members are henceforth referred to as consumers (or interchangeably as agents). The natural resource is assumed to grow according to the standard model of logistic growth, thus the model is relevant to a wide range of resources which includes fisheries, forests, vegetation, foliage, saffron and so on. While Manzoor et al. (2016) give a detailed introduction and exposition of the model, a focus is placed on a society with two agents, where the agents may either be individual consumers or groups of consumers. Here we extend the model further to include an arbitrarily large number of agents. This makes it possible to incorporate the underlying social network, where consumption of a single agent is influenced only by those agents that are socially connected to her. The resultant model bears much similarity with well known models of opinion formation (Friedkin (2006)), which are derived by the same psychological principles and have been studied extensively by the controls community (see Lorenz (2007)). After introducing the basic model, we present a mechanism whereby groups of similar consumers can be aggregated into a single unit provided that certain conditions of homogeneity hold withing the network. Such block models are studied comprehensively for similar processes in sociology (Borgatti et al. (2009)), as they not only simplify the analysis but also utilize the community structure of the network to present a concise description of the society.

The paper is organized as follows. Section 2 presents the basic model and incorporation of the underlying network. Section 3 introduces the block model for societies with prevailing homogeneous communities. Section 4 presents aggregation mechanisms for partially homogeneous and uncertain populations. We conclude in Section 5.

## 2. THE CONSUMER BEHAVIOUR MODEL

The complete model is composed of two parts: the ecological sub-model, and the social sub-model. The ecological sub-model describes the dynamics of the resource, whereas the social sub-model describes the dynamics of consumption effort of each individual in the society.

### *2.1 The Ecological Sub-model*

We assume a renewable stock resource (Perman (2003)), whose quantity at time $\tau$ is represented by $R(\tau)$. In the absence of consumption, the resource grows at the intrinsic growth rate $r$ when it is near depletion, and continues to grow with a decreasing rate until it reaches the carrying capacity $R_{\max}$, at which point it saturates. The resource-stock dynamics are thus given by

$$\frac{dR(\tau)}{d\tau} = r\, R(t) \left(1 - \frac{R(\tau)}{R_{\max}}\right) - \sum_{i=1}^{n} e_i(\tau)\, R(\tau), \tag{1}$$

where $i \in \{1, \ldots, n\}$ and $e_i(\tau)$ is the consumption effort of individual $i$, at time $\tau$, in a society consisting of $n$ individuals. Thus (1) is simply the Gordon-Shaefer model (Gordon (1954)) with the catch coefficient set to unity.

It is important to state here that in the social sub-model to be presented below, the consumption efforts $e_i(\tau)$ can take on both positive and negative values i.e., $e_i(\tau) \in \mathbb{R}$. The interpretation of positive consumption is fairly straight forward i.e., it refers to extraction of the resource. Negative consumption on the other hand, constitutes any measure taken for the sustenance of the resource. This not only includes direct measures like growing trees or breeding fish, but also includes indirect measures such as restoration of soil fertility or restriction of fishing gear. An important implication of negative effort rates which is evident from (1) is that it allows the resource to grow beyond the natural carrying capacity $R_{\max}$. This includes measures such as shifting to intensive agriculture, or adding additional capacity to a fish farm. Manzoor et al. (2016) give detailed interpretations for resource growth beyond carrying capacity as well as negative consumption.

### *2.2 The Social Sub-model*

Here we introduce the cognitive dynamics of the consumers' decision making process. Festinger's social comparison theory (Festinger (1954)) postulates that human beings have the intrinsic drive to evaluate their decisions. They do so by evaluating their decisions against both objective (non-social) information, and social information. Thus in a social-ecological setting, consumers base their use-change decisions on the resource quantity (ecological information) and the consumption of others (social information). What differentiates the decisions of individual consumers is the manner in which they weigh these informations. The ecological information is weighed by $a_i \in (0, +\infty)$, the *attribution* of $i$, where a low value of $a_i$ represents a consumer who associates responsibility for scarcity of the resource with society and a high value represents one who associates the responsibility with nature. Social information is weighed by $s_i \in (0, +\infty)$, the *social value orientation* of $i$, where a low value of $s_i$ represents an extremely non-cooperative individual and a higher value of $s_i$ represents a relatively cooperative individual. Psychological studies (Mosler and Brucks (2003)) find that ecological information is given more importance by individuals attributing scarcity to nature, and cooperative individuals give more importance to social information in order to promote equality in consumption. These effects are depicted by the following dynamic model of consumer behavior.

$$\frac{de_i(\tau)}{d\tau} = a_i(R(\tau) - R_i) + s_i \sum_{j=1}^{n} \omega_{ij}(e_i(\tau) - e_j(\tau)), \tag{2}$$

where $R_i$ is the *scarcity threshold* as perceived by $i$ and $\omega_{ij}$ is the tie strength between $i$ and $j$ with the added constraint that $\sum_j w_{ij} = 1$, and $w_{ii} = 0\ \forall\ i$. Note that $R_i$ is the quantity of the resource above which it is considered abundant by $i$ and below which it is considered scarce. Thus it can also be viewed as a target or set-point that $i$ sets for the resource. A high value of $R_i$ corresponds to a pro-environment individual whereas a low value corresponds to a non-environmental individual. It is also important to realize that this variable depends only on $i$'s perception and does not represent the objective state of the resource.

### 2.3 Non-dimensionalization of the model

Here we reformulate the model through some simple transformations, which not only serves to non-dimensionalize the equations, but also yields parameters which have clearer interpretations in terms of the psychology of the consumers. The transformations occur as follows.

Let $x(\tau)$ be the resource quantity relative to $R_{\max}$, $y_i(\tau)$ be the effort of $i$ relative to the growth rate $r$ and $\rho_i$ be the scarcity threshold for $i$ relative to $R_{\max}$. Thus $x(\tau) = R(\tau)/R_{\max}$, $\rho_i = R_i/R_{\max}$ and $y_i(\tau) = e_i(\tau)/r$. The system can then be formulated as follows

$$\begin{aligned} \dot{x}(t) &= (1 - x(t))x(t) - x(t)\sum_{i=1}^{n} y_i(t), \\ \dot{y}_i(t) &= b_i\Big(\alpha_i(x(t) - \rho_i) - \nu_i \sum_{j=1}^{n} \omega_{ij}\left(y_i(t) - y_j(t)\right)\Big), \end{aligned} \tag{3}$$

where $b_i = \dfrac{a_i R_{\max} + rs_i}{r^2}$, $\alpha_i = \dfrac{a_i R_{\max}}{a_i R_{\max} + rs_i}$, $\nu_i = \dfrac{rs_i}{a_i R_{\max} + rs_i}$ and $t = \dfrac{\tau}{r}$ is the non-dimensionalized time.

Note that the original weights $a_i$ and $s_i$ as they appear in (2) have different dimensions and are incomparable. However the new weights $\alpha_i \in [0,1]$ and $\nu_i \in [0,1]$ are both dimensionless and complementary i.e., $\alpha_i + \nu_i = 1$. Thus $\nu_i$ represents the preference that $i$ places on social information, relevant to the preference placed on ecological information. Therefore, we call $\alpha_i$ and $\nu_i$ as the *ecological relevance* and *social relevance* of $i$ respectively. As described previously, $\rho_i$ determines how pro-environment $i$ is and so we call it the *environmentalism* of $i$. $b_i$ is called the *susceptibility* of $i$ and represents the openness of $i$ to change in her consumption.

### 2.4 Influence and leadership in the underlying network

Here we introduce some notation regarding the network induced by the system (3). While $\omega_{ij}$ represents the strength of the social tie directed from $j$ to $i$, the overall effect of $j$ on $i$'s consumption is given by $b_i\nu_i\omega_{ij}$, the *influence* of $j$ on $i$. The *in-influence* of $i$ is defined as the aggregate influence on the consumption of $i$, from all other members of the network. Conversely, we call the aggregate influence that $i$ exerts on other members of the network as the *out-influence* of $i$. The *net-influence* of $i$ is simply her out-influence minus in-influence and can be viewed as representing $i$'s role in the network as either a *leader* (+ve net-influence), *follower* (-ve net-influence) or *neutral* (zero net-influence).

## 3. AGGREGATION VIA COMMUNITY STRUCTURE AND BLOCK MODELING

In Section 2, the network of the consuming population was presented at an individual level. In this section, we explore the structural effects of the network at two additional levels of abstraction. The homogeneous consumer network allows the abstraction of individual consumptions by viewing the society as a whole. Needless to say, this abstraction comes with a loss of information (in the form

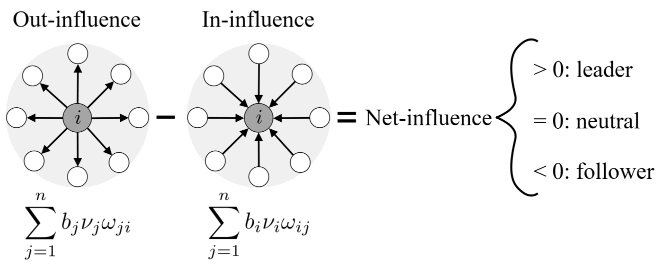



Figure 1. A conceptual illustration of the net-influence of node $i$.

of individual consumptions) and additional constraints (explored in this section) that the network must satisfy. The semi-homogeneous and symmetric semi-homogeneous consumer networks abstract the individual consumptions at the group level, where it is assumed that the society consists of homogeneous communities with certain regularities prevailing in the influences across groups. In what follows, we define the homogeneous, semi-homogeneous and symmetric semi-homogeneous consumer networks and examine the regularities enforced by each on the structure of the consuming population. Together, we call these three constructs the *canonical* consumer networks.

### 3.1 Canonical Consumer Networks and Corresponding Block Models

The consumption dynamics of (3) can be summed over all consumers as follows.

$$\sum_{i=1}^{n}\dot{y}_i = \sum_{i=1}^{n} b_i\alpha_i(x-\rho_i) - \sum_{i=1}^{n}\Big(\sum_{j=1}^{n}\big(\omega_{ij}\, b_i\nu_i - \omega_{ji}\, b_j\nu_j\big)\Big) y_i.$$

In order to complete the aggregation, we require the following lemma

*Lemma 1.* The following condition

$$\sum_{j=1}^{n}\big(\omega_{ij}\, b_i\nu_i - \omega_{ji}\, b_j\nu_j\big) = \beta, \quad \forall\, i \in \{1,\ldots,n\},$$

can hold only for $\beta = 0$.

**Proof.** Assuming that the condition holds true, we can expand the expression as follows

$$\sum_{j=1}^{n}\omega_{ij}\, b_i\nu_i - \sum_{j=1}^{n}\omega_{ji}\, b_j\nu_j = \beta.$$

Summing over all $i$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\omega_{ij}\, b_i\nu_i - \sum_{i=1}^{n}\sum_{j=1}^{n}\omega_{ji}\, b_j\nu_j = n\beta.$$

Switching the index variables for the second term on the R.H.S

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\omega_{ij}\, b_i\nu_i - \sum_{j=1}^{n}\sum_{i=1}^{n}\omega_{ij}\, b_i\nu_i = n\beta,$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\big(\omega_{ij} - \omega_{ij}\big) b_i\nu_i = n\beta \Rightarrow \beta = 0,$$

which concludes the proof. $\square$

Due to Lemma 1, we conclude that in order to express the system in terms of the aggregate consumption $\sum y_i$, the term $\sum_{j=1}^{n}\big(\omega_{ij}\, b_i\nu_i - \omega_{ji}\, b_j\nu_j\big)$ must vanish. This leads to the following definition of our first block model,

*Definition 1.* **Homogeneous consumer network:** A consuming population is said to comprise a homogeneous consumer network if the following conditions hold

i) All agents have uniform attribution $a_i$ and social value orientation $s_i$ i.e., there exist $\mathrm{B}, \mathrm{A}$ and $\mathrm{V}$ such that,
$$b_i\alpha_i = \frac{a_i R_{\max}}{r^2} = \mathrm{B\,A}\ \forall\ i \in \{1,\ldots,n\},\ \text{ and}$$
$$b_i\nu_i = \frac{s_i}{r} = \mathrm{B\,V}\ \forall\ i \in \{1,\ldots,n\}.$$

ii) All agents have uniform environmentalism i.e., there exists $\mathrm{P}$ such that,
$$\rho_i = \mathrm{P}\ \forall\ i \in \{1,\ldots,n\}.$$

iii) There are no leaders or followers in the network i.e.,
$$\sum_{j=1}^{n}\big(\omega_{ji}\, b_j\nu_j - \omega_{ij}\, b_i\nu_i\big) = 0 \quad \forall\, i \in \{1,\ldots,n\}.$$

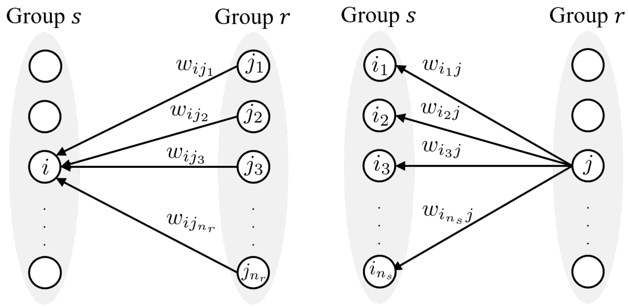
Thus the consumption dynamics of the homogeneous network can simply be represented as
$$\dot{x} = (1-x)x - xY, \quad \dot{Y} = n\mathrm{B\,A}\,(x - \mathrm{P}), \tag{4}$$
where $Y = \sum_{i=1}^{n} y_i$. Thus the system is now represented by a two-dimensional equation which gives a macro-level view of the society as one single unit. It is important to realize here that due to this aggregation, we loose information on the individual consumptions.

We now assume that the network has $m$ non-overlapping consumer groups with populations given by $n_k, k \in \{1\ldots m\}$ respectively, and $n = \sum_{k=1}^{m} n_k$. The groups are classified on basis of their social and ecological relevances, and their environmentalisms. For clarity of notation, let us define the set $N_k = \{k_1, k_2, \ldots, k_{n_k}\}$ to contain the indices of all consumers belonging to group $k$. We then expand (3) as follows.

$$\begin{aligned}
\dot{Y}_1 = \;& \sum_{i\in N_1} b_i\alpha_i(x-\rho_i) \\
& + \sum_{i\in N_2}\Big(\sum_{j\in N_1} b_j\nu_j\omega_{ji}\Big)y_i + \cdots + \sum_{i\in N_m}\Big(\sum_{j\in N_1} b_j\nu_j\omega_{ji}\Big)\,y_i \\
& - \sum_{i\in N_1}\Big(\sum_{j\in N_1}\Big(b_i\nu_i\omega_{ij} - b_j\nu_j\omega_{ji}\Big) + \sum_{j\in N\setminus N_1} b_i\nu_i\omega_{ij}\Big)y_i, \\
& \vdots \\
\dot{Y}_m = \;& \sum_{i\in N_m} b_i\alpha_i(x-\rho_i) \\
& + \sum_{i\in N_1}\Big(\sum_{j\in N_m} b_j\nu_j\omega_{ji}\Big)y_i + \ldots + \sum_{i\in N_{m-1}}\Big(\sum_{j\in N_m} b_j\nu_j\omega_{ji}\Big)y_i \\
& - \sum_{i\in N_m}\Big(\sum_{j\in N_m}\Big(b_i\nu_i\omega_{ij} - b_j\nu_j\omega_{ji}\Big) + \sum_{j\in N\setminus N_m} b_i\nu_i\omega_{ij}\Big)y_i,
\end{aligned}$$

where $Y_k = \sum_{i\in N_k} y_i$. The following definition gives the mechanism for expressing the system completely in terms of the $Y_k$'s

*Definition 2.* **Semi-homogeneous consumer network:** A consuming population is said to comprise a semi-homogeneous consumer network if the following conditions hold




$$\sum_{j\in N_r}\omega_{ij} = d^-_{sr}\ \forall\ i \in N_s \qquad\qquad \sum_{i\in N_s}\omega_{ij} = d^+_{sr}\ \forall\ j \in N_r$$

(a) Pictorial representation of (5) for a general network.

(b) Pictorial representation of (6) for a general network.

Figure 2. Conditions on cross-influence between groups in the semi-homogeneous network

i) There exist $m$ groups in the network, each of which individually fulfill the conditions for the homogeneous consumer network according to Definition 1

ii) The total in-influence of one group on another group should be uniformly distributed across all nodes of the influenced group i.e., for all $i \in N_s$, there exists $d^-_{sr}$ such that
$$\sum_{j\in N_r} b_i\nu_i\omega_{ij} = \mathrm{B}_s\mathrm{V}_s\sum_{j\in N_r}\omega_{ij} = \mathrm{B}_s\mathrm{V}_s d^-_{sr}, \tag{5}$$
where $\mathrm{B}_s$ and $\mathrm{V}_s$ represent the common sensitivity and social relevance of Group $s$ respectively.

iii) The total out-influence of one group from another group should be uniformly distributed across all nodes of the influencing group i.e., for all $j \in N_r$, there exists $d^+_{sr}$ such that,
$$\sum_{i\in N_s} b_i\nu_i\omega_{ij} = \mathrm{B}_s\mathrm{V}_s\sum_{i\in N_s}\omega_{ij} = \mathrm{B}_s\mathrm{V}_s d^+_{sr}. \tag{6}$$

The consumption dynamics of the semi-homogeneous network can thus be written as

$$\begin{aligned}
\dot{Y}_1 = \;& n_1\mathrm{B}_1\mathrm{A}_1(x-\mathrm{P}_1) - \mathrm{B}_1\mathrm{V}_1\Big(n_2 d^-_{12} + \cdots + n_m d^-_{1m}\Big)Y_1 \\
& +\ n_1\mathrm{B}_1\mathrm{V}_1 d^+_{12}Y_2 + \cdots + n_1\mathrm{B}_1\mathrm{V}_1 d^+_{1m}Y_m, \\
\vdots \;& \qquad\qquad\qquad \vdots \\
\dot{Y}_m = \;& n_m\mathrm{B}_m\mathrm{A}_m(x-\mathrm{P}_m) - \mathrm{B}_m\mathrm{V}_m\Big(n_1 d^-_{m1} + \ldots + n_{m-1} d^-_{m(m-1)}\Big)Y_m \\
& +\ n_m\mathrm{B}_m\mathrm{V}_m d^+_{m1}Y_1 + \cdots + n_m\mathrm{B}_m\mathrm{V}_m d^+_{m(m-1)}Y_{m-1},
\end{aligned}$$

where $\mathrm{B}_k, \mathrm{A}_k, \mathrm{V}_k$ and $\mathrm{P}_k$ represent the psychological characteristics of Group $k$. Note that although the system dynamics are expressed completely in-terms of the $Y_k$'s, they still do not appear as a scaled version of (3). For this, we require the following final construct

*Definition 3.* **Symmetric semi-homogeneous consumer network:** A consuming population is said to comprise a symmetric semi-homogeneous consumer network if the following conditions hold

i) There exist $m$ homogeneous groups in the network. Together the $m$ sub-groups fulfill the conditions for the semi-homogeneous consumer network as specified by Definition 2.

ii) The size of all connected subgroups are equal, the resulting implication being that the aggregated out-influences and in-influences as defined by (5) and (6) are equal for each pair of sub-groups, i.e.,

$$d_{sr}^{-} = d_{sr}^{+} = \mathrm{W}_{sr} \text{ for all } s, r \in \{1, \ldots, m\}. \tag{7}$$

The coupled dynamics for the resource and consumption can now be expressed for the symmetric semi-homogeneous network as follows

$$\begin{aligned} \dot{x} &= (1-x)x - x\sum_{i=1}^{m} Y_i, \\ \dot{Y}_i &= n_i \mathrm{B}_i\, \mathrm{A}_i\, (x - \mathrm{P}_i) - \mathrm{B}_i \mathrm{V}_i \sum_{j=1}^{m} \mathrm{W}_{ij}(Y_i - Y_j), \end{aligned} \tag{8}$$

where $i \in \{1, \ldots, m\}$. Note that the homogeneous consumer network given by (4) is a special case of the symmetric semi-homogeneous network with $m = 1$. The utility of the block model (8) is that it presents a picture of the society at a community level which can aid in informed decision making at the macro-level. The block model includes the effects of community sizes and the linkages between them, similar to a regular network except that the nodes are not individual consumers but groups of consumers with similar characteristics. Obtaining (8) in the same form as the original model (4) shows that the block model and aggregation scheme are scalable, and also carries forward the interpretation of system variables and parameters.

### *3.2 Bonding and Bridging in the Symmetric Semi-homogeneous Network*

Here we discuss interpretations of the lumped social ties $\mathrm{W}_{ij}$ in the symmetric semi-homogeneous consumer network defined by (8). From the definition of $\mathrm{W}_{ij}$ we have that $\sum_j \mathrm{W}_{ij} = 1$, for all $i \in \{1, \ldots, m\}$, however $\mathrm{W}_{ii} \geq 0\ \forall\, i$, as opposed to $\omega_{ii}$ which is zero by definition. $\mathrm{W}_{ii}$ is defined by

$$\mathrm{W}_{ii} = 1 - \sum_{\substack{j=1 \\ j \neq i}}^{m} \mathrm{W}_{ij} = \sum_{s \in N_i} \omega_{rs}\ , \text{ for any } r \in N_i.$$

Thus $\mathrm{W}_{ij}$ gives the tie-strength between members of group $i$, which represents the *bonding* capital of the group. Conversely $\mathrm{W}_{ij}, i \neq j$ represents the *bridging* capital between the groups. Both these quantities are complementary i.e., a group that has strong bonding will have weak bridging and vice versa which also agrees with observations of real-world communities (see Easley and Kleinberg (2010)).

It is also interesting to note that the aggregated model (8) allows the possibility of isolated nodes in the network, which did not exist in the original model. Such a node represents a group that has no bridging capital and maximum bonding capital.

## 4. AGGREGATION MECHANISMS FOR NON-CANONICAL CONSUMER NETWORKS

Here we look at aggregation mechanisms for populations that do not satisfy the requirements for the canonical networks presented in Section 3. We first consider communities that have no leaders or followers as given by condition (iii) of Definition 1. We call the underlying network for such populations a *self-directed* network. We show for such networks, that one can construct a block-model without any inaccuracies by intelligently selecting the values for the lumped parameters. We then consider aggregation when accurate information about the consumer characteristics may not be available in a self-directed network.

### *4.1 An Aggregation Mechanism for Self-directed Networks*

Assume a self-directed network, i.e., a population of $n$ consumers in which there are no leaders or followers. Thus the following condition holds

$$\sum_{j=1}^{n} \big(\omega_{ji}\, b_j \nu_j - \omega_{ij}\, b_i \nu_i\big) = 0 \quad \forall\, i \in \{1, \ldots, n\}. \tag{9}$$

Note that no assumptions have been made yet on the ecological weights $\alpha_i$ and the environmentalisms $\rho_i$. In what follows, we show that the aggregate consumption dynamics for such a network can be described similar to (4) if the choices for the aggregate environmentalism and ecological weight are made appropriately. If (9) holds, then the aggregate consumption evolves as

$$\begin{aligned} \dot{Y} &= \sum_{i=1}^{n} b_i \alpha_i (x - \rho_i) = \sum_{i=1}^{n} b_i \alpha_i x - \sum_{i=1}^{n} b_i \alpha_i \rho_i, \\ &= n \left( \frac{1}{n} \sum_{i=1}^{n} b_i \alpha_i \right) \left( x - \sum_{i=1}^{n} \frac{b_i \alpha_i}{\sum_{j=1}^{n} b_j \alpha_j}\ \rho_i \right), \\ &= n\, \hat{\mathrm{B}} \hat{\mathrm{A}} \big(x - \hat{\mathrm{P}}\,\big), \end{aligned}$$

where the lumped parameters $\hat{\mathrm{B}}$, $\hat{\mathrm{A}}$ and $\hat{\mathrm{P}}$ have been chosen such that

$$\hat{\mathrm{B}}\hat{\mathrm{A}} = \frac{1}{n} \sum_{i=1}^{n} b_i \alpha_i; \quad \hat{\mathrm{P}} = \sum_{i=1}^{n} \frac{b_i \alpha_i}{\sum_{j=1}^{n} b_j \alpha_j}\ \rho_i. \tag{10}$$

Thus $\hat{\mathrm{B}}$ and $\hat{\mathrm{A}}$ are chosen such that the lumped product $\hat{\mathrm{B}}\hat{\mathrm{A}}$ is the average of the individual products $b_i \alpha_i$ and the lumped environmentalism $\hat{\mathrm{P}}$ is chosen as a convex combination of the individual environmentalisms $\rho_i$. It is easy to see that this combination is equal to the steady state value of the resource and so $\hat{\mathrm{P}} = \bar{x}$, where $\bar{x} = \lim_{t \to \infty} x(t)$ (if the limit exists). Given this aggregation mechanism, the coupled resource and aggregate consumption dynamics are expressible as

$$\dot{x} = (1-x)x - xY, \quad \dot{Y} = n\, \hat{\mathrm{B}}\, \hat{\mathrm{A}}\, (x - \hat{\mathrm{P}}), \tag{11}$$

where $\hat{\mathrm{B}}, \hat{\mathrm{A}}$ and $\hat{\mathrm{P}}$ are chosen in accordance to (10).

### *4.2 Aggregation for Self-directed Populations with Unknown Characteristics*

Consider a self-directed consumer network whose dynamics are given by (11). If information of the population characteristics are not available, one may aggregate the system as

$$\dot{\tilde{x}} = (1 - \tilde{x})\tilde{x} - \tilde{x}\tilde{Y}, \quad \dot{\tilde{Y}} = n\, \tilde{\mathrm{B}}\, \tilde{\mathrm{A}}\, (x - \tilde{\mathrm{P}}), \tag{12}$$

where $\tilde{\mathrm{B}}$, $\tilde{\mathrm{A}}$ and $\tilde{\mathrm{P}}$ are chosen at the aggregator's discretion. One can then define an error vector $e = [e_x\, e_Y]^T$ such that

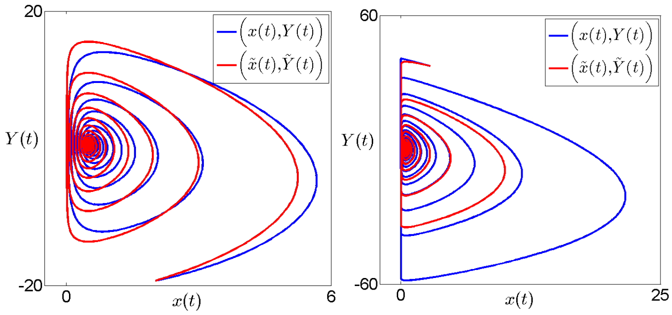



Figure 3. Trajectories of original and approximated aggregated systems when $\tilde{\mathrm{P}} = \bar{x}$, for two different initial conditions. Here $n = 100$, $b_i = 1 \;\forall\; i$, $\omega_{ij} = 1/n \;\forall\; i \neq j$, $\rho_i \sim \mathrm{U}(0, 1)$ and $\rho_i \sim \mathrm{U}(0, 1)$.

$e_x = x - \tilde{x}$ and $e_Y = Y - \tilde{Y}$. From (12) and (11), the steady state error is expressible as follows

$$\bar{e}_x = \bar{x} - \tilde{\mathrm{P}}, \quad \bar{e}_Y = (1 - \bar{x}) - (1 - \tilde{\mathrm{P}}),$$

which shows that if $\tilde{\mathrm{P}}$ is chosen to be equal to $\bar{x}$, then the steady state error is zero regardless of the choice of $\tilde{B}$ and $\tilde{\mathrm{A}}$. Figure 3 shows the behaviour of this approximation scheme when the initial conditions are known perfectly. It can be seen that the error exists only during the transient.

This suggests that for systems at steady state for which information on the resource is readily available, one may aggregate such that $\tilde{\mathrm{P}} = \bar{x}$ and choose any estimate for parameters $\tilde{\mathrm{B}}$ and $\tilde{\mathrm{A}}$. In the event of any disturbances, it is then guaranteed that the error in aggregation will exist only during the transient.

## 5. CONCLUSION

In this paper we have reformulated a recently reported model of natural resource consumption in context of the social network of the consuming population. The model represents the cognitive behavior of human consumers, treated as an integral part of the system which may or may not adapt to changes in the environment. This is in contrast to the conventional method of treating humans as unknown disturbances, exogenous inputs or uncertainty in the system. This paper has extended the basic model by incorporating the underlying social network and studying its structural effects under an aggregation mechanism via homogeneous communities present in the population. The aggregation yields a concise representation of extremely complex networks, which not only makes the analysis tractable, but can also aid in informed decision making at the macro-level. While such block models are frequently employed in practice, in most cases the condition of homogeneity within groups is more of an approximation than ground reality. Building on the techniques described in this paper, one can quantify the error of such approximations so that an appropriate level of confidence may be attached to the resulting decisions. Needless to say, there exists a multitude of directions for future research, and while we do acknowledge that there remain some questions to be addressed before the model is fully specified in a control-theoretic sense (for instance, global stability, analysis of transient behavior, etc), we view this as an exciting opportunity for research and lay the exposition open to critical examination and investigation of the scientific community.

## REFERENCES


Anderies, J.M. (2000). On modeling human behavior and institutions in simple ecological economic systems. *Ecological Economics*, 35(3), 393–412.

Borgatti, S.P., Mehra, A., Brass, D.J., and Labianca, G. (2009). Network analysis in the social sciences. *science*, 323(5916), 892–895.

Bousquet, F. and Le Page, C. (2004). Multi-agent simulations and ecosystem management: a review. *Ecological modelling*, 176(3), 313–332.

Easley, D. and Kleinberg, J. (2010). Networks, crowds, and markets. *Cambridge Univ Press*, 6(1), 1–6.

Festinger, L. (1954). A theory of social comparison processes. *Human relations*, 7(2), 117–140.

Friedkin, N.E. (2006). *A structural theory of social influence*, volume 13. Cambridge University Press.

Friedkin, N.E. (2015). The problem of social control and coordination of complex systems in sociology: A look at the community cleavage problem. *IEEE Control Systems*, 35(3), 40–51.

Gordon, H.S. (1954). *The economic theory of a common-property resource: the fishery*. Springer.

Hare, M. and Deadman, P. (2004). Further towards a taxonomy of agent-based simulation models in environmental management. *Mathematics and computers in simulation*, 64(1), 25–40.

Jager, W. (2000). *Modelling consumer behaviour*. Rijksuniversiteit Groningen.

Jager, W., Janssen, M., De Vries, H., De Greef, J., and Vlek, C. (2000). Behaviour in commons dilemmas: Homo economicus and homo psychologicus in an ecological-economic model. *Ecological economics*, 35(3), 357–379.

Jager, W. and Mosler, H.J. (2007). Simulating human behavior for understanding and managing environmental resource use. *Journal of Social Issues*, 63(1), 97–116.

Lorenz, J. (2007). Continuous opinion dynamics under bounded confidence: A survey. *International Journal of Modern Physics C*, 18(12), 1819–1838.

Manzoor, T., Rovenskaya, E., and Muhammad, A. (2016). Game-theoretic insights into the role of environmentalism and social-ecological relevance: A cognitive model of resource consumption. *Ecological Modelling*, 340, 74–85.

Mosler, H.J. and Brucks, W.M. (2003). Integrating commons dilemma findings in a general dynamic model of cooperative behavior in resource crises. *European Journal of Social Psychology*, 33(1), 119–133.

Munir, S., Stankovic, J.A., Liang, C.J.M., and Lin, S. (2013). Cyber physical system challenges for human-in-the-loop control. In *Presented as part of the 8th International Workshop on Feedback Computing*.

Nguyen, T.A. and Aiello, M. (2013). Energy intelligent buildings based on user activity: A survey. *Energy and buildings*, 56, 244–257.

Perman, R. (2003). *Natural resource and environmental economics*. Pearson Education.

Schirner, G., Erdogmus, D., Chowdhury, K., and Padir, T. (2013). The future of human-in-the-loop cyber-physical systems. *Computer*, (1), 36–45.