A Model Driven Framework for Risk Mitigation in Irrigation Networks

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Abstract

Irrigation networks, like all water resources face uncertainty in operating conditions due to long term risks of climate change and political uncertainty. In Pakistan, canal operation is hierarchical by design and demands an analytical framework for risk analysis and mitigation that respects this layered management structure. In this paper we have investigated a risk mitigation framework that combines tools from risk analysis with system theoretic tools of predictive control to regulate complex canal infrastructures. The solution involves a hierarchical decomposition of optimization problems and the use of data driven and parametric models. While the framework is theoretical and the some of the data and parameters in the study are fictitious, the simulations assure us that the framework is capable of generating good advice for uncertainty quantification and risk management. We conclude that real-time hydrometry (e.g. of rainfall and canal flows) and systematic model development (of both physical flows and risk models) can be the key towards implementing such a framework with large dividends.

1. Introduction

One of the major causes related to the scarcity and distribution of water in developing world countries is the inefficiency of their irrigation networks. In countries like Pakistan, the irrigation networks are extremely vast and complex, the water sources are few and drying, while the infrastructure is in a constant decay, dating back by decades and centuries. Pakistan's Indus river basin hosts the largest contiguous canal network in the world. Water supply to the Pakistan's agriculture base is mostly provided by this complex network. The system operation is largely inefficient, resulting in intractability of usage, wastage and disputes.

While there is an obvious need for building new decision support systems and upgrading civil infrastructures and control institutions, the basin managers are largely unprepared to tackle the effects of uncertainties in the operation of the canals. There are many long- and short-term risks associated with climate change, political uncertainty, economic factors and decay of infrastructures and institutions. The risks are significant even while basin managers may be practicing sound practices of traditional water management in a "business as usual" scenario. In this paper, we make the case that such uncertainties may be quantified and tackled by equipping the basin managers with a risk-analysis and mitigation framework. The particular problem we wish to tackle is that of distribution of water to various stakeholders while ensuring equity and legal entitlements under physical scarcity. This problem is posed at every level of decision making in the Indus basin, from interpretations of Indus-water treaty and implementation of provincial accords to rotational sharing of water among farmers using classical warabandi. The major source of uncertainty in the model is due to the fluctuations in the source. These factors may include climate related risk factors such as flash floods and impulsive draught-like conditions or uncertainties due to political factors such as difficulties in the interpretation of a treaty. While every accord, treaty and institutional control mechanism caters for distribution strategy under normal conditions, most are silent on ways to tackle impulsive situations of temporary excess and sharply increasing scarcity. Since such impulsive events are predicted to rise due to climate change, an analytical framework is needed to give policy makers explicit guidelines. Most importantly, the basin managers need to understand what to expect. While it is true that climate change will bring unprecedented levels of uncertainty in system operation, there are still ways to quantify and bound it.

Our framework relies on two main ideas. First, we setup a hierarchy of optimization problems, by which we translate the effects of uncertainty at the top due to factors like climate change and political uncertainty propagate down to the lower level such as change in regulation set-points in the operation of canals. At each level we setup appropriate optimization framework that enumerates and links the risk factors with their mitigation actions. Second, we show how to use parametric models of channel flows and real-time telemetry to drive the dynamic optimization problem frameworks in a moving horizon of prediction. While we understand that this is only a framework, and the effort required to calibrate the models and obtain real-time data is a major proposal for infrastructure update, we note that without such decision support systems (hydroinformatics) and data collection driven interventions (hydrometry), the basin managers can not translate the storylines of climate change and political uncertainty into appropriate systems analysis questions for appropriate response.

The rest of the paper is organized as follows. In section 2, we spell the general risks associated with climate change and political uncertainty for water resources in general and Pakistan in particular. Next in section 3, we describe the hierarchical structure of water management in canal networks in the Indus Basin. In section 4, we begin our modeling framework and setup the hierarchical optimization problems in a general setting. Some simulation results which demonstrate how the framework can be used for a typical canal operation scenario are presented in section 5. Finally, in section 6, we give some policy recommendations and conclude the paper.

2. Climate Change, Water, Uncertainty and Risks

2.1. Climate Change and Risks

In this section, we summarize the general risks associated with climate change for water resources in general and canal networks of countries like Pakistan in particular. In various reports [1], [2], [3], it has been projected that human activity is causing a rise in concentration of greenhouse gases, in turn causing higher levels of solar energy trapped in the atmosphere. Summer and winter temperatures in Asia are projected to rise by 0.1-0.2 degrees per decade. This will cause an intensification of the hydrological cycle and hence increase the risks of floods and drought. Dry seasons will become dryer due to greater evaporation and results in desertification. Rainy seasons wetter due to greater precipitation and may results in higher number of tropical storms, hurricanes and cyclones. There will also be a negative effect on the general availability and quality of fresh water. Temperatures will also risk melt of glaciers, which will further increase flood risks in the rainy seasons. For many countries, the rising sea levels will cause coastal erosion, flooding of wetlands, salt water intrusion and salinisation of groundwater. In human societies, this will have several direct and indirect effects. Projected water related climate change effects will impact agricultural practices and risk food security. There can be health impacts, decrease in economic activity and even conflict over water resources. In this uncertain future, mitigation and adaption strategies are critical for future planning. These include steps like rainwater harvesting, groundwater recharge, water transfer schemes, restoration of aquatic ecosystems, building of reservoirs, decreasing water demand, increasing use efficiency, drought-resistant crops, changing cropping patterns, repair and maintenance of irrigation infrastructure, improvement of urban water and sanitation, improved flood protection and flood forecasting. There can be several barriers in mitigation and adaptation programs due to lack of realization of the problems, separated strategies and lack of agreements amongst stakeholders, institutional barriers, lack of participation and financing. One of the major barriers is uncertainty in projections and lack of frameworks to tackle the risk, which is the subject of this paper.

2.2. Climate Change Hydrological Projections for Pakistan

For Pakistan, water related climate change studies have been summarized in a study by WAPDA as follows [4]. The average temperature over Pakistan will increase in the range of $1.3 - 1.5\,^oC$ by 2020 and $2.5 - 2.8\,^oC$ by 2050. It is projected that monsoon rainfall could increase by 5% -50% in 60 years, with an associated doubling in the frequency of high intensity rainfall events. Variability in monsoon rains will intensify the frequency and severity of floods and droughts. Upstream intrusion of saline water in Indus delta may adversely affect coastal ecosystems (such as mangrove forests) and fishery. Rising sea levels will pose a threat to coastal city populations (in particular Karachi) along with an increase in cyclonic activity due to higher sea surface temperatures. There are conflicting studies on the fate of Indus basin glaciers. River flow trends from 1966-2002 can be summarized as follows. There has been a significant increase in summer flows from the Karakoram eastern tributaries. This indicates warmer summers and glacier recession. On the other hand, a significant drop in summer flows has been noted from the Karakoram western tributaries. This indicates that summers are cooling, while the glaciers may be stable or even growing in the western region.

2.3. Uncertainty Quantification in Climate Change Analysis

In most water resource planning and climate change studies, uncertainties are quantified using two methods: Probabilistic methods and scenario planning [5]. Sources of uncertainty include factors such as natural hydrological variability, imprecision in model parameters, demographic projections, transition to new technologies and human factors in decision making. Probabilistic methods are used in water systems analysis to represent randomly varying parameters. Probability distributions might be used to quantify a model parameter whose value is unknown but for which a realistic range of possibilities can be constructed from expert opinion or can be extracted from past data. The performance of a risk mitigation strategy is measured in terms of probability functions calculated from input distributions. The result of this analysis can be viewed as an overall assessment of risk in decision making.

Scenario based planning is another method that is in wide use in climate change studies. First, several plausible scenarios of potential future conditions are defined. Strategies are then evaluated under these different scenarios to determine the one that is most robust. In stead of distributions, scenario trees or scenario matrices are constructed to evaluate alternate plans.

3. Hierarchical Structure of Canal Operation in the Indus Basin

3.1. Hierarchical Structure

Water from the Indus river and its affiliates provides water for a majority of the country's 170 million population. Most of the water is used in agriculture, utilizing up to 60% of the total resources [6]. The Indus originates in the northern mountainous regions of the country and flows north-south, passing through the plains of Punjab and Sindh, and finally discharging into the Arabian Sea with a combined annual average volume of 178 Billion Cubic Meters (BCM). As of 2010, there are 4 main reservoirs, 19 barrages and 43 main canals in the Indus Basin Irrigation System (IBIS) with another 12 inter-river link canals and 107,000 smaller watercourses. These are manmade structures added to the main river system over the past decades and centuries to enable the delivery of water to fertile areas not directly irrigable by rivers or natural rainfall. The role of the link canals is to fill shortage in the natural flow of some rivers due to international accords. Each main canal further branches into secondary (branch) canals, distributory canals and tertiary watercourses (khalas). The combined length of the canals is estimated at roughly 60,000 km. Secondary watercourses and field channels further add approximately 1.5 million km to the network length. A network flow diagram of these infrastructures and their interconnection has been depicted in Figure 1. Water usage for irrigation in this system is temporally divided into two seasons: Kharif (April-Sept) and Rabi (Oct-March).

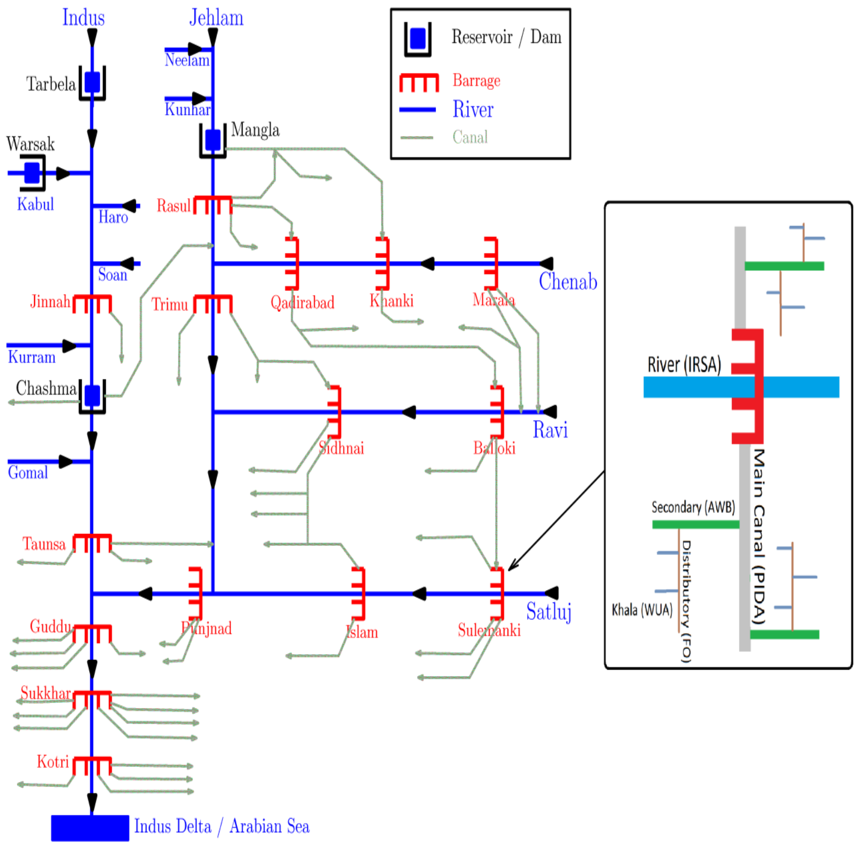

Figure 1: Indus Basin Irrigation System.

As evident from Figure 1, the system is hierarchical at various levels. Therefore, institutional and operational mechanisms have evolved (or developed) at every level to manage the distribution of water in the network. These mechanisms control allocation of water for:

1. River systems via international and provincial accords (e.g. Indus Water Treaty)
2. Canal command systems via regional and provincial regulatory bodies (e.g. PIDA bodies)
3. Farmer water rights via formal and informal institutions. (e.g. warabandi)

We explain below this institutional hierarchy in order to understand both the current structure. The nomenclature used below maps to the IMT institutional reforms of 1990s. Some of the proposed institutions (e.g. AWB) are still ineffective and synonymous with the structure of irrigation departments before reforms. However, we will keep the nomenclature proposed by the reforms and concentrate instead on the operational hierarchy that arises in either interpretation.

3.1.1. Level 1: River and Storage Systems under Indus River System Authority (IRSA)

Indus River System Authority (IRSA) was created in 1992 to implement the Water Apportionment Accord agreed among the Provinces in 1991. At the time of the Accord the Indus Basin system comprised of the Tarbela dam on the main Indus River, the much smaller Mangla dam on the Jhelum River, the network of link canals constructed under the Indus Basin Replacement Works program as a part of the Indus Water Treaty, and the system of barrages to divert water into the canals, some of which had existed since the 19th century[7].

3.1.2. Level 2: Canal Commands under Provincial Irrigation and Drainage Authority

IRSA manages and allocates water to the four provincial irrigation departments. The irrigation department in each province prepares a provincial irrigation demand on a 10-day basis and passes these demands to the IRSA which then looks to make releases from the three major reservoirs (Tarbela, Mangla and Chashma)[8]. Once IRSA allocates water, the PIDAs are thenceforth responsible for distributing that water internally within the canal commands under their jurisdiction. The PIDAs supply canal water to farmers, and manage, operate and maintain the entire irrigation network except the tertiary canals that are maintained by the farmers. The Punjab Irrigation and Drainage Authority for instance consists of 22 main canal systems and 13 barrages/headworks, with an aggregate length of 23000 miles with off-taking capacity of 1,30,000 cusecs.

3.1.3. Level 3: Branch Canal Command under Area Water Boards (AWBs)

Area Water Boards are modeled as financially self- sufficient entities at the canal command levels with functions similar to a utility company. These are responsible for the irrigation and drainage management of the main canal system, including bulk water supplies to the head of the distributaries. At the time of writing of this paper, no AWB is operational in Punjab, although the FOs (explained below) that make up the AWB's are operational in many districts.

3.1.4. Level 4: Distributory Canals under Farmers Organizations (FO)

The farmer organization (FO) is an elected body of farmers that represent farmers at the minor/ distributaries level and form an interface between the farmers and all the upper tiers. The functions of FOs include operation and management of the irrigation and drainage infrastructure, equitable distribution of water in their designated areas of representation and collection of abiana from the farmers. A percentage of the dues collected is kept by the FOs to cover their operational costs[9].

3.1.5. Level 5: Watercourse level and Warabandi

At the tertiary level (khalas), water is managed by Khal Panchayat (WUA). At this level, water is supplied in a rotational method called Warabandi. The term literally means "turns" (wahr) which are "fixed" (bandi). Warabandi is a rotational method for equitable allocation of the available water in an irrigation system, by turns fixed according to a time roster, specifying a day, time and duration of supply to each irrigator. It has been practiced in Pakistan and Northern India for more than 125 years and covers a total area of 24 million hectares. The warabandi system provides a continuous rotation of water in which one complete cycle of rotation generally lasts 7 days. The duration of supply for each farmer is proportional to the size of the farmer's landholding to be irrigated within the particular watercourse command area[10].

At this level of water distribution, usually the time sharing water distribution schedule provided by the District Irrigation Department is strictly followed, however, there are instances where farmers need to deviate from the schedule to fill the gap between water demand and supply. Albeit illegal, this method is adopted to dispose off surplus water in order to save crops or to compensate for evaporative losses, increased cropping intensities, changes in cropping patterns and expansion of arable land within the canal command[11].

4. Mathematical Model and Risk Mitigation Framework

In this section we will develop a mathematical model of an irrigation network followed by a model driven framework for risk mitigation. We will assume a hierarchical structure of canal operation as described in the previous section. In this approach, the decisions at the higher level influence the parameters at the lower levels and the inflows from a higher level to a lower level are considered as given boundary conditions.

4.1. Mathematical Modeling of an Irrigation Network

The primary objective of an irrigation network at any level in the hierarchical structure is water distribution from reservoirs to sinks (water consumers). To develop a mathematical model, we assume an irrigation network comprising of $\hat{N}$ water reservoirs and $\hat{M}$ sinks connected through a network of irrigation canals. Any irrigation canal can be further divided into several canal reaches for the purpose of controlling and regulating the water flows. The point between any two canal reaches can be considered as an intermediate node. The amount of water that enters an intermediate node is exactly equal to the amount of water leaving this node. In other words, this node is different from a reservoir or sink node since it cannot store or consume water.

Let V^R denote the set of all reservoir nodes, V^S denote the set of sink nodes and V^I denote the set of intermediate nodes in the network. Let $V = V^R \sqcup V^I \sqcup V^S$. We can model this network as a graph with nodes and edges. The nodes are connected by edges, indicating the presence of water channels. The edges are weighted by the flow of water in the canal reach or channel, modeled by a flow matrix $\mathbf{f} \in \mathbb{R}^{|V| \times |V|}$. An element f_{ij} of matrix $\mathbf{f}$ models a directed flow from node i to j in the network. Each flow f_{ij} is a function of time t and space variable x i.e. $f_{ij}(t, x)$. Let, L_{ij} denote the length of the canal reach (m) between node i and node j. Let A_{ij} denote the cross-sectional area of the reach between nodes i and j. The surface area of node i is denoted by A_i^s. Let, h_i denote the water level (m) at node i. Similarly, let N_i^d represent the set of immediate downstream neighbors of node i and N_i^u as the upstream neighbors. We now model open canal reach and water reservoirs.

4.1.1. Open Canals

The flows and water levels in an open canal reach can be described by the Saint-Venant equations. Estimates of flow rate or water level at certain locations in the channel are obtained by using a set of partial differential equations that define the conservation of mass and momentum along this channel. These equations allow us to compute the flow and water level as a function of space and time. However, in this model the spatial variations in lateral and transverse directions are neglected and the flow in an open channel can be approximated as a one dimensional process along the longitudinal direction denoted by variable x (i.e., in the direction of flow). An open channel reach between nodes i and j in the irrigation network can be modeled as,

$$\frac{\partial f_{ij}(t,x)}{\partial x} + \frac{\partial A_{ij}}{\partial t} = q_{ij}^{lat}, \quad \forall i,j \in V, \tag{1}$$

$$\frac{\partial f_{ij}(t,x)}{\partial t} + \frac{\partial}{\partial x}\left(\frac{f_{ij}^2(t,x)}{A}\right) + gA_{ij}\frac{\partial h_{ij}}{\partial x} + \frac{gf_{ij}(t,x)|f_{ij}(t,x)|}{c^2 r_{ij}^f A_{ij}} = 0, \quad \forall i,j \in V, \tag{2}$$

where, $g = 9.81m/s^2$ is the gravitational acceleration, c is the Chezy friction coefficient $(m^{1/2}/s)$, r_{ij}^f is the hydraulic radius, q_{ij}^{lat} is the lateral inflow per unit length (m^2/s) and variable t denotes time (s). Eq (1) is a mass balance equation which ensures conservation of water volume while eq (2) is a momentum balance equation.

4.1.2. Reservoirs

Water reservoirs e.g. dams can be modeled by the mass balance equation only. The water level h_i is calculated as a function of the inflows and outflows.

$$A_i^s \frac{dh_i(t)}{dt} = f_i^{in}(t) - f_i^{out}(t), \quad \forall i \in V^R, \tag{3}$$

where, $f_i^{in}(t) = \sum_{k \in N_i^u} f_{ik}(t, L_{ik})$ represent the inflows in the reservoir from the upstream neighbors of node i and $f_i^{out}(t) = \sum_{k \in N_i^d} f_{ki}(t, 0)$ denote the outflows from the reservoir towards its downstream neighbors.

4.2. Optimization Problem for Water Management

We now develop an optimization problem for water management. Let T denote the time horizon of the optimization problem. The objective of the optimization problem is the minimization of cost function. Let $\mathcal{J}(t, \beta)$ denote the time varying cost function defined for the network (β is the optimization variable vector e.g. set points, flows etc. An example of cost function is given in (17)). Let $C_i(t)$ denote the available water in reservoir node i at time t.

Similarly, let $D_i(t)$ denote the water demanded by sink node i. Then we have the following optimization problem for such a dynamic water network.

$$\min_{\beta} : \quad \int_0^T \mathcal{J}(t,\beta)dt, \tag{4}$$

subject to the following constraints,

$$\frac{\partial f_{ij}(t,x)}{\partial x} + \frac{\partial A_{ij}}{\partial t} = q_{ij}^{lat}, \quad \forall i,j,x \in [0,L_{ij}], \tag{5}$$

$$\frac{\partial f_{ij}(t,x)}{\partial t} + \frac{\partial}{\partial x}\left(\frac{f_{ij}^2(t,x)}{A}\right) + gA_{ij}\frac{\partial h_{ij}}{\partial x} + \frac{g f_{ij}(t,x)|f_{ij}(t,x)|}{c^2 r_{ij}^f A_{ij}} = 0, \forall i,j,x \in [0,L_{ij}], \tag{6}$$

$$A_i^s \frac{dh_i(t)}{dt} = f_i^{in}(t) - f_i^{out}(t), \quad \forall i \in V^R, \tag{7}$$

$$f_i^{out}(t) \le C_i(t), \quad i \in V^R, \forall t, \tag{8}$$

$$f_i^{in}(t) \ge D_i(t), \quad i \in V^S, \forall t, \tag{9}$$

$$f_i^{in}(t) = f_i^{out}(t), \quad i \in V^I, \forall t, \tag{10}$$

$$f_{ij}(t,x) \ge 0, \quad i,j \in V, \forall t, x \in [0,L_{ij}], \tag{11}$$

$$h_i(t) \ge 0, \quad i \in V, \forall t. \tag{12}$$

In this problem, the first three constraints (5), (6) and (7) conserve the flow in the open channels and water level in the reservoirs according to the model. Constraint (8) ensures that the total outflow from any reservoir node is less than its capacity. Constraint (9) ensures that the total inflow into into any sink node is less than the demanded value. Constraint (10) means that the inflow is equal to outflow at any intermediate node in the network. Constraints (11) and (12) are the non-negativity constraints on the flows and reservoir heights. The solution of this problem will give the optimal values of the optimization variable vector β such that the cost function is minimized and all the constraints are satisfied. However, this problem is very difficult to solve due to the presence of non-linear partial differential equations as constraints.

We therefore develop a discrete time linear model of the irrigation canal. In this model, instead of using the Saint-Vennat equations, canal reach dynamics are modeled by a first order system plus a delay. This model is also known as integrator-delay model in the literature [12], [13]. A pure delay is placed in series with a discretized integrator. In this model, the change in the water level at each intermediate node (between two canal reaches) between any two time slots is given by the following equation,

$$A_{ji}\left(h_i(t+1) - h_i(t)\right) = \Delta t\left(f_i^{in}(t - t_i^d) - f_i^{out}(t)\right), \forall i \in V^I, j \in V^R \sqcup V^I, \tag{13}$$

where t_i^d is the delay or time required for a change in the inflow of intermediate node to cause a change in water level at the intermediate node. Note that this delay occurs because water has to travel the length of the reach to arrive at the intermediate node. In this discrete model, Δt is the length of time interval and the time horizon T is divided into N intervals i.e. $T = N\Delta t$. Similarly, the discrete time model of water reservoir is given as,

$$A_i^s\left(h_i(t+1) - h_i(t)\right) = \Delta t\left(f_i^{in}(t) - f_i^{out}(t)\right), \quad \forall i \in V^R. \tag{14}$$

Equations (13), (14) therefore simplify the dynamics of an irrigation network into water level changes at the reservoirs and intermediate nodes. The non-linear Saint-Vennat equations which model canal reach dynamics at every point in space and time can therefore be removed from the optimization problem.

4.2.1. Setting an Objective Function for the optimization problem

The discrete time objective function in optimization variable vector β can be defined as,

$$\min_{\beta} \quad \sum_{t=0}^{N} \mathcal{J}(t, \beta). \tag{15}$$

It is often convenient to consider a distributed objective function of the following form,

$$\mathcal{J}(t, \beta) = \sum_{i \in V} \mathcal{J}_i(t, \beta_i), \tag{16}$$

where, $\mathcal{J}_i(t, \beta_i)$ is the objective function of node i at time t and β_i denote the optimization variables of node i. The objective function $\mathcal{J}_i(t, \beta_i)$ can be set by the designer depending on the desired objectives. This approach divides the optimization problem into several individual optimization problems. Some typical control objectives $\mathcal{J}_i(t, \beta_i)$ in an irrigation network can be the following:

- to regulate downstream water levels
- to minimize cost on water supply
- to make sure that right amount of water is available at the right place
- to ensure that operational spills are avoided

For example if the objective is to regulate downstream water levels and h_i^{ref} denote the reference height for node i, then the objective function of such an optimization problem can be given as,

$$\mathcal{J}_i(t, h_i) = \left(h_i(t) - h^i_{ref}\right)^2, \tag{17}$$

where, $\beta_i = h_i$. An objective function can also be defined as a weighted combination of several objectives.

4.3. Risk Mitigation Framework

In a hierarchical distribution network, the decisions at the higher levels influence the parameters at the lower levels. In such networks, we introduce uncertainties at a higher level and then consider possible risk mitigation actions along with their impact at the lower level parameters. We explain this approach through a generic two level network shown in 2. In this figure, the higher level is assumed to be exposed to several external risks e.g. due to climate change etc. At the higher level, the probability of each risk is evaluated and certain risk mitigation actions are planned. Again, each risk mitigation action is assumed to have an associated cost. The risk mitigation actions generally reduce the risk exposure, however, due to the associated costs, the decision makers have to decide whether some action is necessary or not. Therefore, an appropriate risk exposure minimization optimization problem has to be developed at the higher level. Risk mitigation actions modify the lower level parameters e.g. inflows or reference heights at the canal reaches. At the lower level, an optimization problem is then required for flow regulation and opening / closing of the gates to maintain e.g. the reference heights. It should be noted that in this framework, we can set different time scale at different levels. For example, at the higher level, time scale $t = 1$ may represent a month or a year. At the lower level, however $t = 1$, may be a day or an hour.

In the next sections we discuss the higher and lower level optimization problems in the risk mitigation framework.

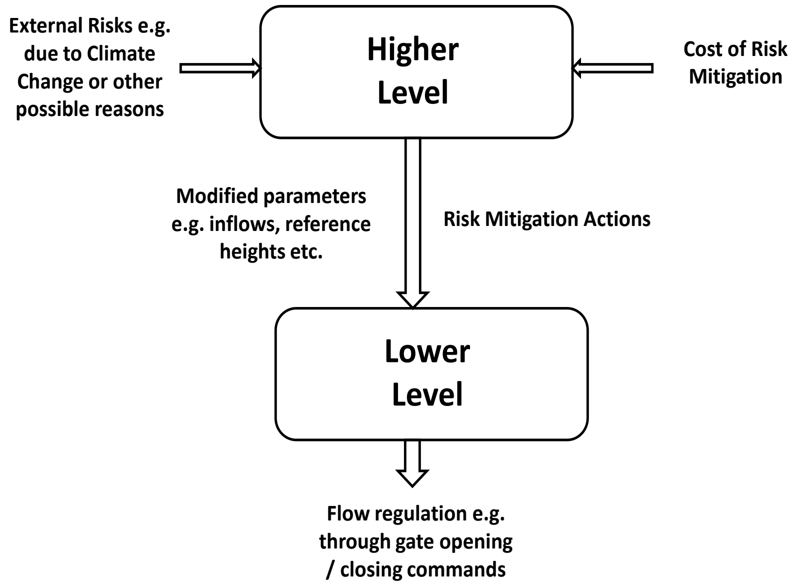

Figure 2: Risk mitigation framework.

4.3.1. Higher level Optimization problem in Risk Mitigation Framework

Let $\{R_i\}_{i=1}^{M}$ denote the set of possible risks. Some possible risks due to climate change can be the following:

- increased water levels (floods),
- reduced water levels (droughts),
- change in policies (water accord),
- change in water demands by provinces.

Let $\{Z_i\}_{i=1}^{K}$ denote the set of parameters that are liable to suffer from risks. Some examples of the parameters which are liable to change are given below:

- variation in reference levels,
- economic costs,
- water demands,
- time delays.

Any risk or group of risks can change the value of some or all of these parameters e.g. floods can have an impact on the economic costs and reference levels. Each risk can be associated to a set of possible mitigation actions. Few examples of risk mitigation actions can be the following:

- construction of dams,
- modification of reference levels,
- appropriate monitoring of rivers, dams, glaciers etc,
- insurance policy,
- lining of irrigation canals.

Risk model must clearly identify the parameters, risks, their probabilities, and the impact on each parameter. Once risks and their impact on parameters are clearly identified, the next step is the design of a strategic risk mitigation plan to reduce the impact of these risks. Let $\{A_i\}_{i=1}^{J}$ denote the set of risk mitigation actions. Each risk mitigation action is a function of three elements i.e.

$$A_i = \{u_i, F_i, G_i\}$$

, where:
u_i: is the decision variable for mitigation action A_i (can take binary values 0 or 1),
F_i: set of function that determines the risk impact reduction on each parameter,
G_i: extra cost as a result of mitigation action.
Once risk model is properly defined, at each time instant t, the objective at the higher level is to determine appropriate risk mitigation actions i.e. return the values of decision variables u_i. Depending on the actions, new parameter values for the lower level will be calculated.

In order to develop an optimization problem at the higher level we define risk exposure. Risk exposure is defined for each risk taking into account all the previous information about risks. At time t, parameter Z_j's exposure to risk R_i can be defined as [12],

$$RE_i^j(t, u_i) = p_i(t)\left(I_i^j - \sum_{a=1}^{J} \mu_i^a F_a^j(u_i)\right) + \sum_{a=1}^{J} \mu_i^a G_a^j(u_i)$$

, where,
$p_i(t)$ is the probability of risk i at time t,
I_i^c is the initial impact of risk i on parameter j,
μ_i^a is an indicator function: $\mu_i^a = 1$ if risk i is mitigated by mitigation action a while $\mu_i^a = 0$ otherwise,
$F_a^j(u_i)$ risk impact reduction function for parameter j as a result of control action a,
$G_a^j(u_i)$ extra cost of mitigation action a for parameter j
. At any time t the objective at the higher level is to minimize the risk exposure over a finite time horizon N. The objective function can be written as,

$$\mathcal{J}_{HL}(t, u_i) = \min_{u_i} \sum_{i=1}^{M} \sum_{k=1}^{N} \sum_{j=1}^{J} \left[\widehat{RE}_i^j(t+k, u_i) \right]^2, \tag{18}$$

where, $\widehat{RE}_i^j(t+k, u_i)$ is the predicted risk exposure at time $t+k$ (index i is for risk, k for time and j for parameter). Furthermore, we can also associate weights to different parameters j. For example, if some parameter is more important then its weight can be set at a higher value. If $p_i(t)$ is known for all time slots, then depending on the control actions, predicted risk exposure can be easily calculated.

4.3.2. Lower level Optimization problem in Risk Mitigation Framework

Risks generally can be mitigated by taking appropriate mitigation actions. These actions then change the reference values e.g. reference heights for the lower level optimization. Let, h^i_{newref} denote the new reference height for node i then we can modify the lower level objective function $\mathcal{J}_i(t, h_i)$ as,

$$\mathcal{J}_{LL} = \min_{h_i} \quad \sum_{t=0}^{N} \sum_{i \in V} \left(h_i(t) - h^i_{newref} \right)^2, \tag{19}$$

subject to constraints (8), (9), (10), (11), (12), (13), (14). Note that depending on the actual irrigation network some constraints might be removed from the optimization problem. For example, if there are no water reservoirs in some considered irrigation network then the corresponding constraints can be removed from the optimization problem. The lower level optimization problem results in the regulation of water flows and determines appropriate control actions e.g. opening and closing of gates which might be installed at various canal reaches. The output can be different depending on the reference heights as well as the objective functions.

In the next section we will discuss the solutions of the higher and lower level optimization problems.

4.4. Solution of the optimization problem

We now have two optimization problems one at the higher level (18) and another one at the lower level (19). In the following we discuss their solution.

4.4.1. Solution of the higher level optimization problem

Higher level optimization problem (18) can be solved using any mathematical tool (Matlab or ILOG CPLEX) if the variables are too many otherwise with small number of variables all possible outcomes can be enumerated and decision variables with minimum cost can be determined. The solution of this problem therefore determines the optimal risk mitigation actions with minimum cost required for canal operations.

It should be noted here that risk identification, impact and probability of each risk, appropriate risk mitigation actions, cost of each risk mitigation action, etc. are required for successful implementation of this framework. It is therefore up to the basin manager to quantify these parameters from meteorology and economics studies (if not already known). In the framework, at every sampling instant, the cost which will incur without taking any mitigation action and the cost which will incur by execution of mitigation actions can be compared. Each risk mitigation action is executed only if its probability to occur exceeds a certain threshold (which again can be set by the basin manager). Once an action has been taken its corresponding risks wont be assessed again until the effect of the action expires. The frequency of risk assessment and mitigation action can be different for different risks and can also be determined by the basin manager. For example, to mitigate the effect of seepage losses, irrigation canal linings can be reassessed and renewed after every six months. On the other hand, in rainy or drought season, reference heights of water in canal reaches can be modified on daily basis.

4.4.2. Solution of the lower level optimization problem

The decisions made at upper level effect the lower level optimization problem i.e change the reference heights of the water levels in canal reaches. The objective at the lower level is then to regulate water height h_i around the modified set points. The flow regulation problem can be modeled as a control problem and an appropriate controller can be designed to obtain the solution. For a large irrigation network with several canal reaches, distributed control is desirable. Model Predictive Control (MPC) is an optimal control strategy which is based on the explicit use of a model to predict the process output at future time instants and can also be implemented in a distributed manner. In order to develop an MPC, a state space model of flow dynamics (13) is required. To develop a state space model we identify the disturbances that effect the inflows and outflows according to the following equations;

$$f_i^{in}(t - t_i^d) = Q_{in,i}(t - t_d) + q_{in,i}(t), \tag{20}$$

where, $Q_{in,i}$ is the upstream inflow of water and $q_{in,i}$ represent the disturbances e.g. inflows due to rainfall or small input drains etc. Similarly,

$$f_i^{out}(t) = Q_{out}(t) + q_{in,i}(t), \tag{21}$$

where, $Q_{out,i}$ is the downstream outflow of water and $q_{out,i}$ denote the known off take outflows for agricultural use by farmers. The variables $Q_{in,i}$, $q_{in,i}$, $Q_{out,i}$ and $q_{out,i}$ can be understood from Figure 3.

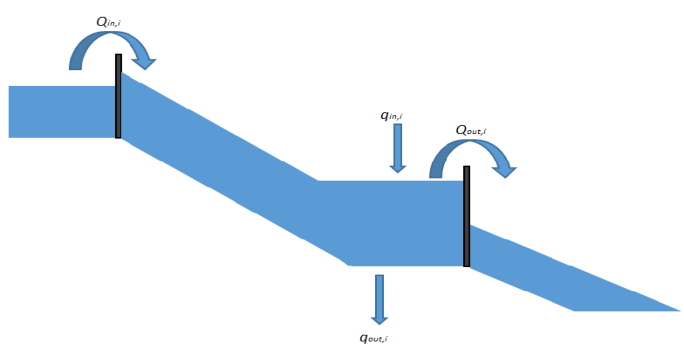

Figure 3: Inflows and outflows in a canal reach.

The flow dynamic equation (13) can now be written as,

$$A_{ji}(h_i(t+1) - h_i(t)) = \Delta t \left(Q_{in,i}(t - t_d) + q_{in,i}(t) - Q_{out,i}(t) - q_{out,i}(t)\right). \tag{22}$$

From this equation we can now develop the following state space model for MPC;

$$\mathbf{x}_i(t+1) = \mathbf{A}_i \mathbf{x}_i(t) + \mathbf{B}_i \mathbf{u}_i(t), \tag{23}$$

$$\mathbf{y}_i(t) = \mathbf{C}_i\mathbf{x}_i(t) + \mathbf{D}_i\mathbf{u}_i(t), \tag{24}$$

where, $\mathbf{x}_i(t) \in \mathbb{R}^{n_{x_i}}$ are the local states,$\mathbf{u}_i \in \mathbb{R}^{n_{u_i}}(t)$ are the local inputs,$\mathbf{y}_i(k) \in \mathbb{R}^{n_{y_i}}$ are the local outputs and $\mathbf{A} \in \mathbb{R}^{n_{x_i} \times n_{x_i}}$,$\mathbf{B} \in \mathbb{R}^{n_{x_i} \times n_{u_i}}$,$\mathbf{C} \in \mathbb{R}^{n_{y_i} \times n_{x_i}}$, $\mathbf{D} \in \mathbb{R}^{n_{y_i} \times n_{u_i}}$ determine how the different variables influence the local states and outputs of the system. Interested readers are referred to [12], [13],[14],[15] for the details on developing this state space model for MPC. Once an appropriate state space model is developed, distributed MPC control tools which are widely available e.g. in Matlab can be used to obtain the optimal values of the optimization variables h_i. Please note that at each time slot t MPC provides a complete set of control actions for next $t+1, \ldots N$ time slots. However, only the current control action $\mathbf{u}_i(t)$ is used and remaining values are discarded. At the start of next time interval, MPC is used again to determine the new control action.

In the next section we will demonstrate the application of the proposed model driven framework through a case study.

5. Case Study

In this section we will do a case study to demonstrate how our model driven risk mitigation framework can be used for irrigation canal network management. In our framework, there are several parameters which have to be defined by the basin manager e.g. type of risk, probability of each risk, risk mitigation action etc. In this case study we will consider three risks; floods, drought and reduction in anticipated water share (according to some political agreement e.g. provincial water accord of 1991) due to some political changes. Occurrence of floods and droughts can be linked to climatic changes while reduction in anticipated water share can be due to purely political reasons. Each risk can occur with a certain probability. Probability of floods and droughts can be obtained either from previous meteorological data or meteorology (MET) department can provide predictions to the basin manager and predict the year to be drought year or flood year or maybe a normal year. In our simulations, we will consider three scenarios:

- **Scenario 1**: Normal Year Prediction: Assuming that the MET department predicts it to be a normal year. In a normal year, the probability of floods and droughts is low.
- **Scenario 2**: Flood Year Prediction: Assuming that the MET department predicts it to be a flood year. In a flood year the probability of

drought is negligible while there is a low probability of this being a normal year.

- **Scenario 3**: Drought Year Prediction: Assuming that the MET department predicts it to be a drought year. In a drought year the probability of floods is negligible while there is a low also probability of this being a normal year.

For political risks, we assume a constant value of 0.01 i.e. 1% chance of deviations from the promised water share. Each risk will have an economic impact. The impact however can be reduced by taking some mitigation actions. The impact of floods can be reduced by incrementing the storage capacity of reservoirs and modifying the reference levels. Similarly effect of droughts can be also be reduced by the same set of actions and further by reducing the seepage losses through renewing the canal lining. In our case study, we assume that the reduction in water share due to political events cannot be mitigated. The risks and their corresponding mitigation actions is shown in Figure 4.

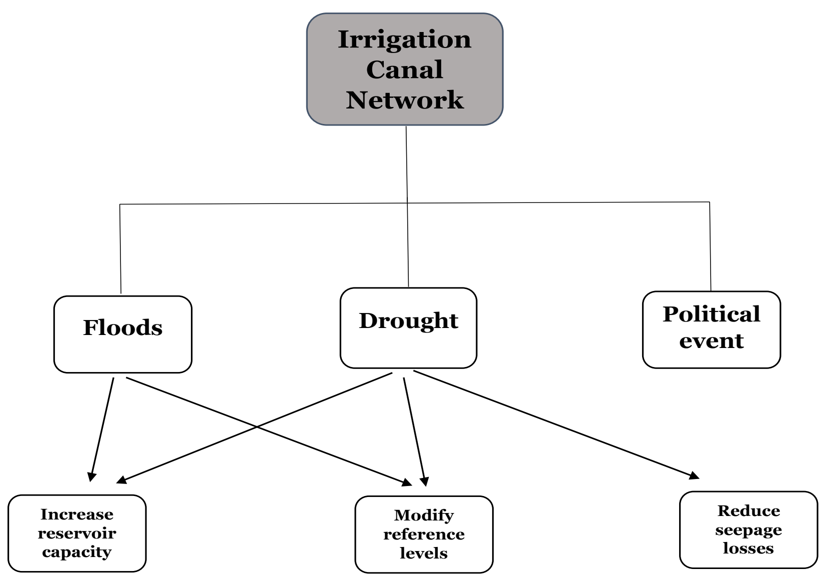

Figure 4: Risks and their associated mitigation actions in our case study.

According to the expression of risk exposure in our framework, the basin manager has to quantify the initial impact of each risk. Based on a mitigation action (or a given combination of mitigation actions) the initial impact can be reduced by α (we will call α the risk mitigation factor in the remaining discussion). For example, increasing the storage capacity by a certain amount (e.g. by building a new dam), might decrease the initial impact of floods by 50% giving us a value of $\alpha = 0.5$. In our case study, we will therefore, change the value of α and see how it effects the cost. As mentioned earlier, there is also a cost associated with some mitigation action. For example, increasing storage capacity or renewing the canal lining comes at a price. However, reference water levels can be easily modified without inuring much cost. The frequency of each mitigation action is decided by the basin manager. In our case study, we assume that we can modify the reference water levels on daily basis and the frequency of remaining actions is once a year.

The initial impact of flood in our case study is associated with the amount of rainfall. On the other hand, the impact of drought is associated with the water levels measured in the canal reaches. Therefore, in order to appropriately quantify these impacts, real time data is required. Generally, monthly mean rainfall data is readily available. However, for our simulations we require daily rainfall values. To generate these values, we randomly set the number of rainy days in each month. The months which receive low rainfalls, typically has less rainy days compared to months which receive more rainfall. Then on each rainy day of each month, we randomly generate the rainfall values such that the mean of the generate values match the given monthly mean rainfall data. The estimated rainfalls for our case study is shown in Figure 5. We can observe that in the monsoon months, there are more rainy days and the amount of rainfall is also high compared to other months.

The water level in canal reaches is also required on a daily basis. In this study, we have used an actual data set recorded for a distributory canal in Hakra Branch canal command area (southern Punjab). The data was obtained in an (on-going) project by our group that concerns the instrumentation of distributory canals in the Indus basin [16, 17]. The original data was recorded at every 10 minutes and is not available for some months of the year. We have down sampled the data to a daily value and copied the missing data with existing data. As verified by experts, this does not change the qualitative properties of the data-set for this particular study. This data is given in Figure 6. We can see that the water level in the canal increases when there is inflow and it decreases when the farmers draw water for their agricultural needs or when the canal does not receive water from its upstream

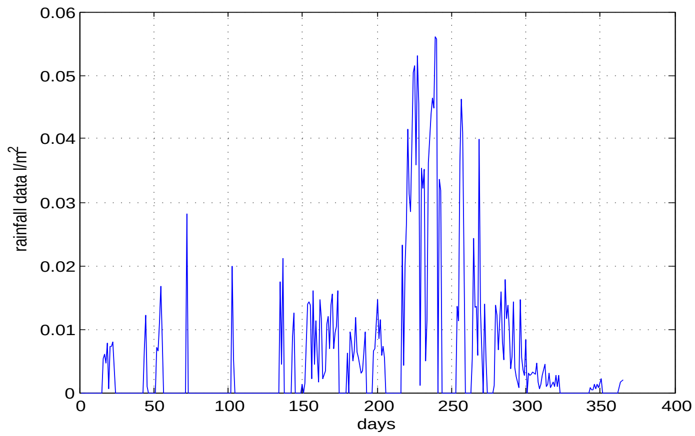

Figure 5: Estimated daily rainfall data for our case study.

nodes.

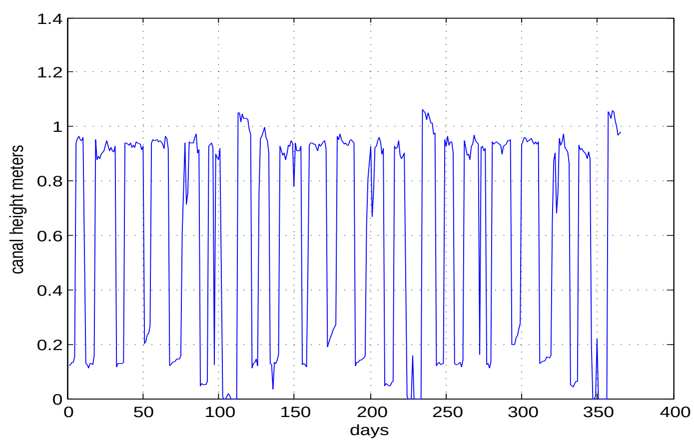

Figure 6: Real time canal data for our case study.

5.1. Simulation Results

In this section, we present simulation results for the upper level and lower level optimization problems.

5.1.1. Upper level optimization problem

For the upper level optimization problem we consider three scenarios as discussed above. Let, P_n, P_f and P_d respectively denote the probability of normal year, flood year and a drought year.

5.1.2. Scenario 1: Normal Year Prediction

In these simulations, we assume that $P_n = 0.8$, $P_f = 0.1$ and $P_d = 0.1$. The probability of floods and drought is assumed to be 10%. In Figure 7 we vary the value of α and study the resulting impact on the overall accumulated cost. When the value of α is 0.3, taking the mitigation action increases the overall cost. This is due to the fact that the cost of mitigation action itself is greater than the cost reduction this action can provide. Moreover, since we have assumed a normal year, therefore, the probability of flood and drought is low and taking the mitigation action only increases the overall cost. When we increase the value of α to 0.6, still the overall cost remains high as compared to the case when we do not take any mitigation action. However, when we increase α to 0.9, we can see that since the mitigation action can remove the initial impact by 90%, therefore, there is a reduction in the overall cost. Thus, basin manager has to carefully consider when mitigation action should be taken. Moreover, mitigation actions should be such that they can greatly reduce the initial impact (more than 60% to justify the mitigation action).

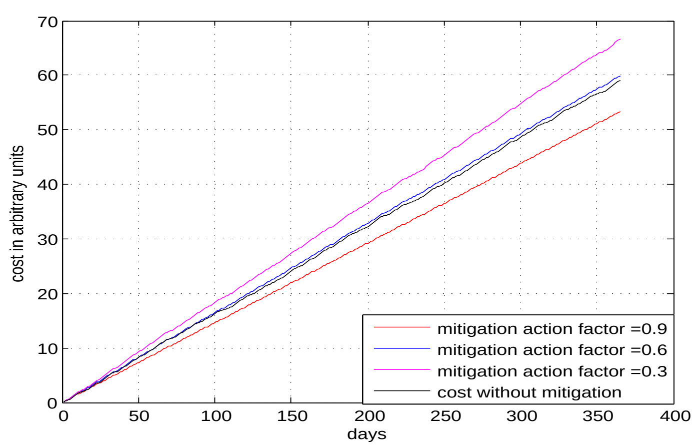

Figure 7: Impact of risk mitigation factor α on the overall accumulated cost in normal year prediction scenario: $P_n = 0.8$, $P_f = 0.1$ and $P_d = 0.1$.

5.1.3. Scenario 2: Flood Year Prediction

In these simulations, we assume that there is a high probability of floods and very low probability of drought. We assume $P_f = 0.7$, $P_d = 0.01$ and

$P_n = 0.29$. In Figure 8 we again consider the impact of risk mitigation factor on overall accumulated cost. Since there is a high probability of floods, we can see that if we do not take any mitigation action, the resulting cost is very high. Even, a low impact mitigation action e.g. $\alpha = 0.3$ can now result in significant savings. On the other hand, if the mitigation actions are high impact, then the resulting reduction in accumulated cost is very promising.

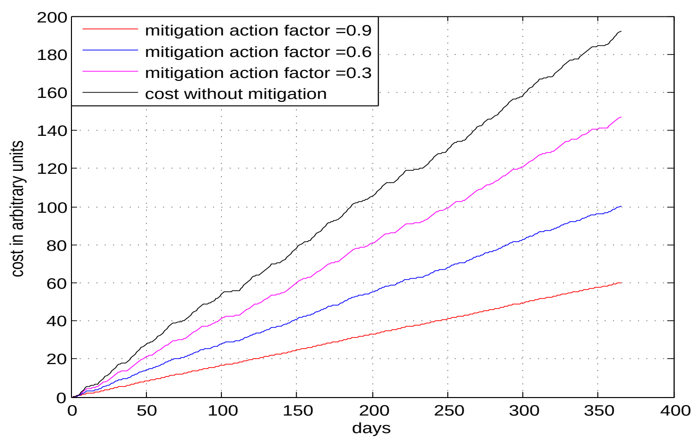

Figure 8: Impact of risk mitigation factor α on the overall accumulated cost in flood year prediction scenario: $P_n = 0.29$, $P_f = 0.7$ and $P_d = 0.01$.

5.1.4. Scenario 3: Drought Year Prediction

In the drought year prediction scenario we assume a high probability of drought and a very low probability of flood. We assume, $P_d = 0.7$, $P_f = 0.01$ and $P_n = 0.29$. In Figure 9, we study the impact of risk mitigation factor on the overall accumulated cost. From this figure, we can that we can reduce the overall accumulated cost by taking a mitigation action. However, the impact of risk mitigation factor is minimal. This shows that mitigation action is very important regardless of its impact. Even a low impact risk mitigation factor can lead to significant cost reduction. Moreover, a high impact risk mitigation action is also not required.

We can notice that these results are highly dependent on the quantification of risk, impacts, rainfall data and water level data in the canal reaches. Thus, considerable effort and infrastructure update is required to calibrate the models and obtain real time data to actually obtain appropriate response from the proposed framework.

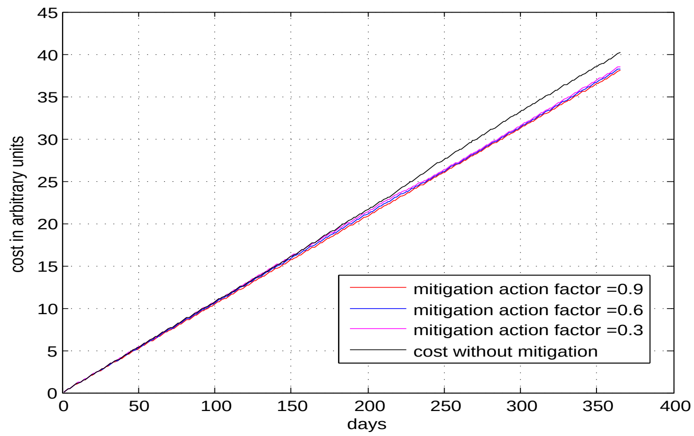

Figure 9: Impact of risk mitigation factor α on the overall accumulated cost in drought year prediction scenario: $P_n = 0.29$, $P_f = 0.01$ and $P_d = 0.7$.

5.2. Lower level problem

At the lower level, we have to regulate the water level around the desired set points in each canal reach by opening or closing the gates by an appropriate amount. We solve the control problem by using Matlab MPC tool. We consider a small network comprising of only two interconnected canal reaches. To make the simulations interesting, we consider actual parameters of Muzaffargarh canal in DG khan, Punjab. The length of first canal reach is 4600 m, the length of second canal reach is 6000 m and the third reach is 7500 m long. The bottom width of all three canal reaches is 50 m. The surface area of the canal reaches is respectively 230,000 m^2, 300,000 m^2 and 375,000 m^2 respectively. The duration of each time slot is assumed to be 60 seconds which is equal to 1 minute (i.e. $\Delta t = 1$ minute). The time required for water in first canal reach to travel from upstream to downstream is assumed to be 100 minutes or 100 times slots. Similarly, the time required for water in second canal reach to travel from upstream to downstream is assumed to be 130 minutes or 130 time slots and for the third canal reach it is taken to be 160 minutes or 160 time slots . The total simulation duration is assumed to be 1600 minutes. The reference height is assumed to be 0.9 m for all the canal reaches and we also assume that initially the water level is exactly equal to the defined set point. In our simulations, we assume that there is a gate at the upstream and downstream of each canal reach. The gate at the upstream of each reach will be referred to as the input gate while the gate at the downstream will be called output gate. Note that the input gate of canal reach 2 in our network is the output gate of canal reach 1. Furthermore, it is also assumed that canal reach 1 is connected to a water reservoir which can provide as much water as required for canal operation. Similarly, canal reach 2 is connected canal reach 3 whose output gate is connected to a water sink which can consume as much water as required for regulation.

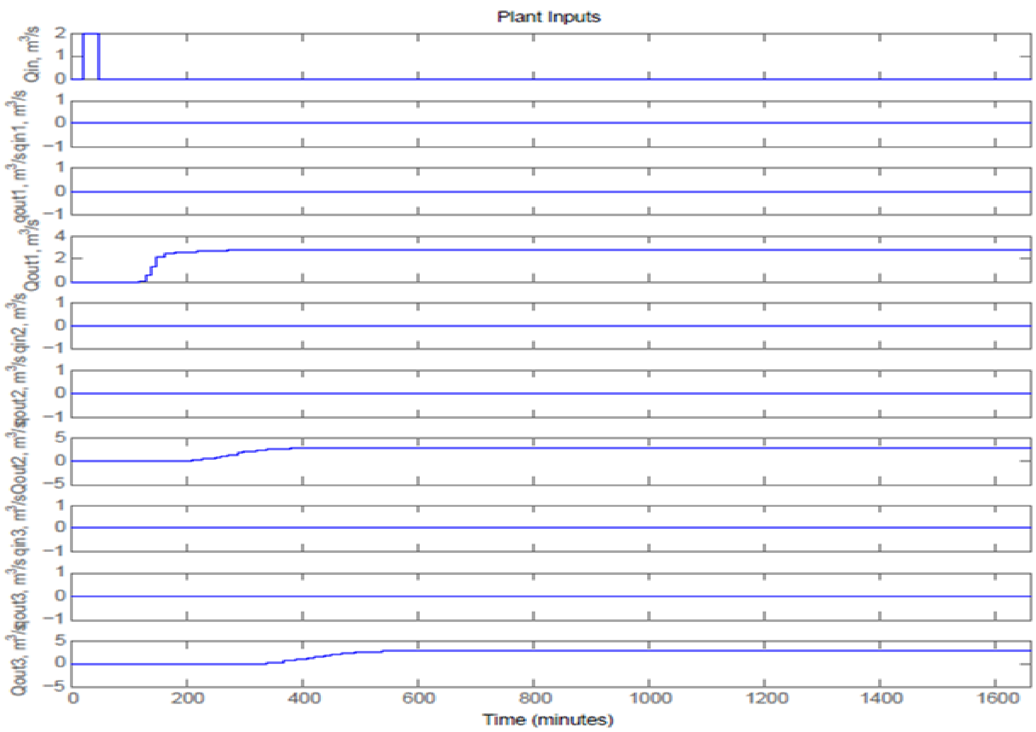

Figure 10: Inflows and Outflows in canal reaches by opening and closing of gates by MPC to maintain reference water levels. Input gate of canal reach 1 is opened for about 0.5 hour and there are no external disturbances.

In Figure 10, we demonstrate how the MPC regulates the water levels to maintain reference heights. In these simulations, we assume that there are no external disturbances $q_{in,i}$ and $q_{out,i}$ (i.e. there is no rainfall and no off takes by the farmers). We disturb the water level in canal reach 1 by opening its input gate for 0.5 hours from 16 till 46 minute point on the time axis in the figure (which indicates minutes). The flow rate of incoming water is 2 m^3/s (which is roughly 70 Cusecs). This increases the water level in canal reach 1 and the MPC opens the output gate at 110 minute point to maintain

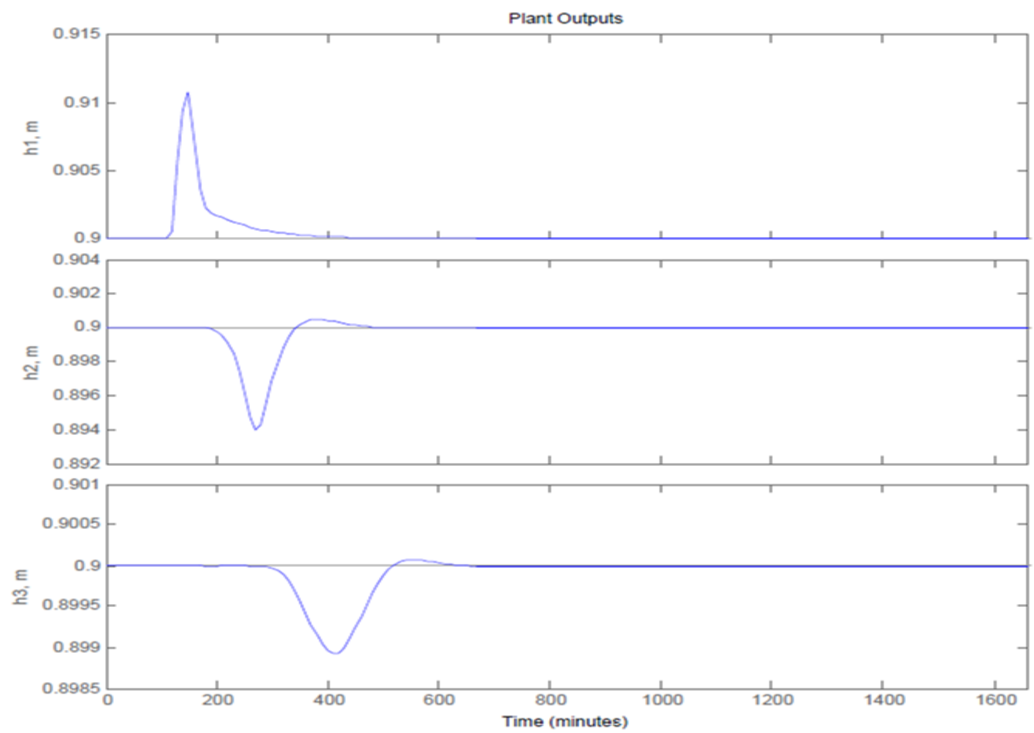

Figure 11: Water level in canal reaches is shown. Water level varies due to opening and closing of gates. Objective of MPC is to maintain at level at 0.9 meters.

the reference height. When the gate is fully open, the outflow increases and reaches a maximum value. The gate remains fully open to allow maximum discharge. Since the water discharged from canal reach 1 enters canal reach 2, therefore, we can also observe that MPC opens the output gate of canal reach 2 around 200 minute point once sufficient water has entered this reach to activate the controller and same way canal reach 3 input gate opens up at around 330 minute point . These simulations demonstrate the opening and closing of gates by MPC as required to maintain reference water levels. In Figure 11 the corresponding outputs i.e the height of water in canals can be seen. The simulations clearly show how the canals have rejected disturbances and maintained heights at 0.9 m level.

6. Conclusions

6.1. Policy Recommendations for Basin Managers

In this section we conclude the paper, highlight some future directions and give a policy brief for basin managers. Policy Recommendations for Basin Managers This work supports the following recommendations and guidelines for water managers and canal operators.

1. **Embrace Uncertainty**: Like all water resources, irrigation canal networks in Pakistan are facing an uncertain future.The presence of uncertainty in the dynamics of water resources such as irrigation canal networks must be recognized, quantified and planned against. With a projected increase in the frequencies of floods and droughts and a rise in political uncertainty, events previously considered as unlikely are increasingly becoming a part of the norm. In particular, for canal operations, services such as equitable and timely water distribution cannot be run in a business as usual scenario.
2. **Tackle Risks Systematically**: There exist computational frameworks to model, identify and mitigate sources of uncertainty as risks to the operation of an irrigation network. The frameworks can equip the basin managers with decision support to play out various scenarios and plan appropriate mitigation actions. One such framework has been presented in this paper.
3. **Invest in Basin Models and Telemetry**: Such frameworks heavily depend on data collection and parametric models. While, this requires a significant investment in time and resources to calibrate parameters and setup real-time data collection, there is a significant payoff in this investment, even for studies outside the scope of this study.
4. **Canal Operation is Hierarchical**: The canal operation problem is hierarchical, and naturally decomposes into a hierarchy of optimization problems. The decisions taken at an upper level can be translated into a change in set-points at the lower level. In particular, the seasonal flow division problem at the canal command level can be translated to a change in regulation scheme at the distributory or watercourse level.
5. **Play Scenarios**: The data- and model-driven optimization framework can help generate scenarios for business as usual (normal year) and extreme events (droughts, floods and political uncertainty.

6. **Small Mitigation in Extreme Events Pays High**: The simulated scenarios educate us that a mitigation action is very important regardless of its impact. Even a low impact risk mitigation factor can lead to significant saving. Moreover, a high impact risk mitigation action is also not required.
7. **Mitigation Action Timing is Critical**: The simulated scenarios tell us that, even in a normal year, the basin manager has to carefully consider when mitigation action should be taken. Moreover, mitigation actions should be such that they can greatly reduce the initial impact.

6.2. Contributions

1. A computational framework suitable for analyzing risks from climate change and uncertainty in a hierarchical management structure for irrigation canal networks.
2. Successful translation of high-level decisions for risk mitigation to low-level changes in canal operation for flow regulation.
3. Use of real-time data from meteorology and canal hydrometry to create a moving horizon of predictive decision making.
4. Numerical solution of predictive control problem for set-point regulation in canal operation for timely and equitable water delivery.
5. Simulation of scenarios under normal and extremal events such as floods, droughts and political uncertainty with a systematic analysis of risk mitigation strategies.

References

[1] Julie Wilk, Hans-Bertil Wittgren, et al. Adapting water management to climate change. *Adapting water management to climate change*, 2009.

[2] BC Bates et al. Climate change and water: Technical paper of the intergovernmental panel on climate change. *IPCC Secretariat: Geneva*, 2008.

[3] C Cabot. Climate change and water resources, 2007.

[4] WAPDA. Wapda official position on climate change.

[5] *Climate Change Handbook for Regional Planning, Appendix C, Canada-Department of Water Resources.* 2011

[6] Asim R Khan, M Kaleem Ullah, and Saim Muhammad. Water availability and some macro level issues related to water resources planning and management in the indus basin irrigation system in pakistan, 2002.

[7] IRSA. Indus river system authority (2010) project management and policy implementation unit. the ministry of water and power, government of pakistan.

[8] Government of Pakistan. National water policy: Water resources of pakistan availability and requirement, page 3 of section 4.

[9] Government of Punjab. Punjab irrigation and drainage authority (2007) scheme for transfer of irrigation management - farmers organizations in punjab.

[10] DJ Bandaragoda. *Design and practice of water allocation rules: lessons from wara- bandi in Pakistan's Punjab*, volume 17. IWMI, 1998.

[11] M. N. Asghar and R. Wahaj. Farmers' management responses to the gap between supply and demand of canal water in a large scale irrigation system. *Proceedings of the workshop on Irrigation Advisory Services and Participatory Extension in Irrigation Management. Montreal, Canada.*, 2002.

[12] Ascensin Zafra-Cabeza, JM Maestre, Miguel A Ridao, Eduardo F Camacho, and Laura Snchez. A hierarchical distributed model predictive control approach to irrigation canals: A risk mitigation perspective. *Journal of Process Control*, 21(5):787-799, 2011.

[13] T. Keviczky R.R. Negenborn, P.-J. van Overloop and B. De Schutter. Distributed model predictive control of irrigation canals. *Networks and Heterogeneous Media,* 4(2):359-380, June. 2009.

[14] T Barjas Blanco, Patrick Willems, PK Chiang, K Cauwenberghs, Bart De Moor, and Jean Berlamont. Flood regulation by means of model predictive control. In *Intelligent Infrastructures*, pages 40-437. Springer, 2010.

[15] PJ Van Overloop, RR Negenborn, B De Schutter, and NC van de Giesen. Predictive control for national water flow optimization in the netherlands. In *Intelligent infrastructures*, pages 439-461. Springer, 2010.

[16] Abubakr Muhammad Waqas Ahmad Zahoor Ahmad, Ehsan U. Asad and Arif Anwar. Development of a low-power smart water meter for discharges in indus basin irrigation network. *Springer Communications in Computer and Information Science (CCIS),* Volume 366, 2013, pp 1-13., 2013.

[17] Zahoor Ahmad and Abubakr Muhammad. Design, calibration and performance of a low-power wireless sensor node for open channel flows. *39th Annual Conference of the IEEE Industrial Electronics Society (IECON), Vienna, Austria,* 2013.