Feedback in the Quantum World: A Control Engineer’s View of the Quantum Information Processing

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Outline

• Motivational examples
• Quantum control problem
• Peculiarities of quantum control
• Role of information in control problems
• Information theoretic approach to quantum control
• Observations
Cavity Quantum Electrodynamics

- Atom trap in ultra-low loss optical cavity
- Strong coupling between atom and radiation
- Use real-time feedback to cool atom to ground state of quantized potential
- Measure phase of leaking light
- Adjust laser to control depth of optical potential
- Removes KE from atomic motion

Source. Mabuchi et al. 2002
Control Problems In General

- **Objective:** Enhance system performance, despite noise and parameter uncertainty

- Process (Noisy)
- Control inputs
- Disturbances
- Controller
- Actuators

- Open Loop
  - No measurement

- Closed Loop
  - Measurement based feedback
  - Measurement noise
Open Loop Control In Quantum Systems

• Process time evolution (model)
  – Schrodinger Equation for wave function
  – Liouville Equation for density matrix
  – More complicated dynamics of open systems

• Open Loop control
  – Design time evolution for an initial state to target a final state

• Closed Loop
  – Measurement based feedback
  – Measurement noise
Types of Quantum Feedback Control

- Measurements in the quantum world can be tricky (not found in classical control)
- Direct or indirect coupling between control and process (not found in classical control)
Information theory and Quantum Dynamics

- Evolution of a quantum system $Q$ (and environment $E$)
  
  $$\rho = U_{QE} (\rho_0 \otimes \rho^E) U_{QE}^\dagger$$
  $$\rho^Q = \mathcal{E}(\rho_0) = \sum_i E_i \rho_0 E_i^\dagger$$
  von Neumann entropy $S = \text{Tr} \rho^Q \log \rho^Q$

- Start with a pure state --- end up in a mixed one.
- The von Neumann entropy always increases.
  $$S(Q) = S(\rho^Q) > S(\rho_0)$$

- Question: Can you force the entropy to reduce?
Information theory and Quantum Control

Open Loop Strategy
Global Unitary operation

\[ \rho^C = \sum_i p_i |i\rangle_C \langle i| \]

\[ \rho^{QC} = \rho^Q \otimes \rho^C = \sum_{i,j} p_i \rho^Q_j \otimes |i\rangle_C \langle i| \]

\[ \rho^{QC} \rightarrow U_{\text{open}} \rho^{QC} U_{\text{open}}^\dagger \]

\[ S(Q, C) = S(Q_{\text{out}}, C_{\text{out}}) \leq S(Q_{\text{out}}) + S(C_{\text{out}}) \]

\[ \Delta S_Q^{\text{open}} \equiv S(Q) - S(Q_{\text{out}}) \leq S(C_{\text{out}}) - S(C) \]

Entropy reduction.
Entropy reduction upper bounded by maximum amount of entropy increase of C.
Information theory and Quantum Control

\[ U_{\text{open}} = \sum_i U_i \otimes |i\rangle_C \langle i| \]

\[ \rho_{Q_{\text{out}}C_{\text{out}}} = \sum_{i,j} p_i U_i \rho_j^Q U_i^\dagger \otimes |i\rangle_C \langle i| \]

\[ S(Q_{\text{out}}) = S \left( \sum_i p_i U_i \rho_i^Q U_i^\dagger \right) \geq \sum_i p_i S \left( U_i \rho_i^Q U_i^\dagger \right) = S(Q) \]

\[ \Delta S_Q^{\text{open}} \leq 0 \]

Try another strategy

LOCC (Local Quantum operation, Classical Communication)

One can never reduce entropy this way
Information theory and Quantum Control

Closed Loop Strategy

Use measurement that does not change entropy.

e.g. conventional von-Neumann

Maximum improvement over open loop limited by quantum mutual info obtained by controller!
Information theory and Quantum Control

Closed Loop Strategy 2: LOCC

Maximum entropy reduction exactly equal to quantum mutual info obtained by controller.

This can be thought of as a quantum error correction process.
Where do we go from here?

- So far we have discussed feedback of classical information.
- What about coherent quantum feedback?
- What if there is a limit on how much quantum information can be pumped back?
- Need to know about quantum capacity of quantum channels.
Role of Information in Control

\[ S_{e,d}^2(\omega) = \frac{F_e(\omega)}{F_d(\omega)} \]

\[ \frac{1}{2\pi} \int_{-\pi}^{\pi} \log S_{e,d}^2(\omega) d\omega \geq 2 \sum_{\text{unstable poles}} \log|\text{pole}_i| \]

Stability implies:

\[ \liminf_{k \to \infty} \frac{I(x_0; e^k)}{k} \geq \sum_{\text{unstable poles}} \log|\text{pole}_i| \]

Bode Integral Formula

\[ h(e^k) \geq h(d^k) + I(x_0; e^k) \]

\[ h_\infty(e) \geq h_\infty(d) + \liminf_{k \to \infty} \frac{I(x_0; e^k)}{k} \]

Classical Entropy

\[ H(X) = -\sum_{x \in \mathcal{X}} p_X(x) \log p_X(x) \]
Role of Information in Control

\[ C_f < \infty \quad \text{Finite Capacity Feedback} \]

\[
\frac{1}{2\pi} \int_{-\pi}^{\pi} \min \left\{ \log \left( S_{e,d}(\omega) \right) \right\} d\omega \geq \sum_{\text{unstable poles}} \log |\text{pole}_i| - C_f
\]
What type of capacity?

• No one knows the answer. (to-date)

• Capacity in feedback path is different from classical Shannon capacity of comm. channels.
  – Anytime capacity (noisy channels) [Mitter et al.]
  – Topological feedback capacity (noiseless channels) [Nair et al.]

• Open problem. What is the capacity definition to define feedback stabilization in quantum systems?
Why do we care about capacities?

- **Communication engineer**
  - Noise is ubiquitous
  - What are the physical limits of pumping info in communication channels in the presence of noise?
  - Does the quantum world offer any new limits/surprises?
  - Do quantum limits degrade communication capabilities?

- **Experimental physicist**
  - How sensitive can an apparatus be made?
  - Resolution: a parameter in the Hamiltonian settles to how many final states?

- **Control engineer**
  - How much information needs to be fed back in order to stabilize? (Quantum control)
Conclusions

- Quantum world has peculiar control structures
- Many problems in quantum information can be understood from control perspective
- The control perspective has a nice information theoretic explanation
- There are several open problems and avenues of research in this (young) area.
- Opportunity for engineers, physicists, computer scientists alike.

- Thank you!
References


• Information-Theoretic Limits of Control by Hugo Touchette and Seth Lloyd. arxiv.org/abs/chao-dyn/9905039v1

• Information Theoretical Approach to Control of Quantum Mechanical Systems by Shiro Kawabata. arxiv.org/abs/quant-ph/0409187v1