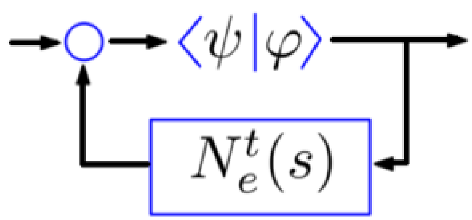
Feedback in the Quantum World: A Control Engineer's View of the Quantum Information Processing



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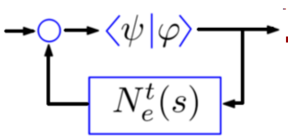
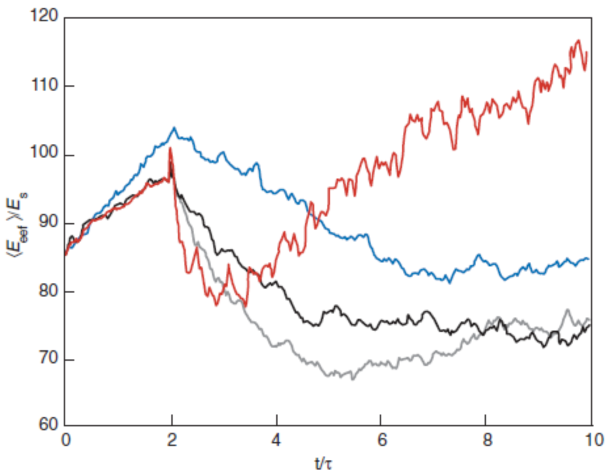
Outline

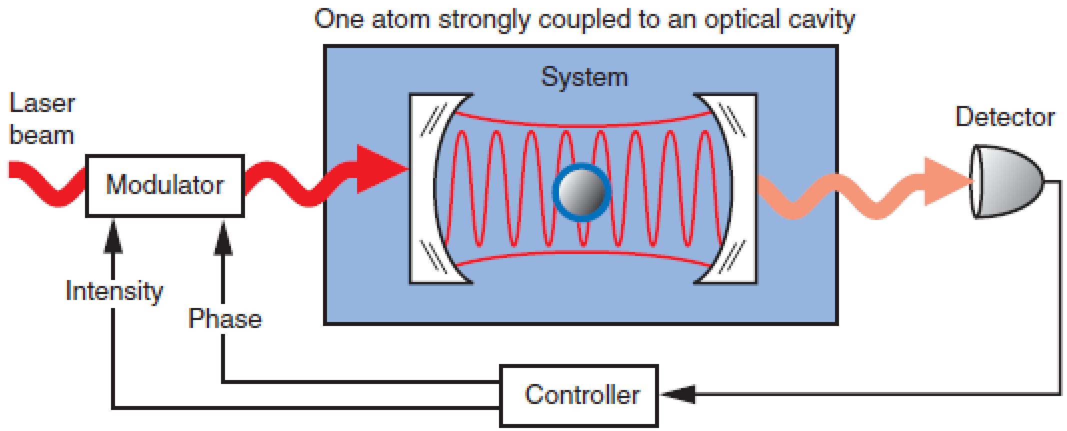
- Motivational examples
- Quantum control problem
- Peculiarities of quantum control
- Role of information in control problems
- Information theoretic approach to quantum control
- Observations



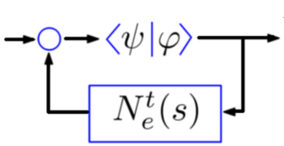
Cavity Quantum Electrodynamics

- Atom trap in ultra-low loss optical cavity
- Strong coupling between atom and radiation
- Use real-time feedback to cool atom to ground state of quantized potential
- Measure phase of leaking light
- Adjust laser to control depth of optical potential
- Removes KE from atomic motion





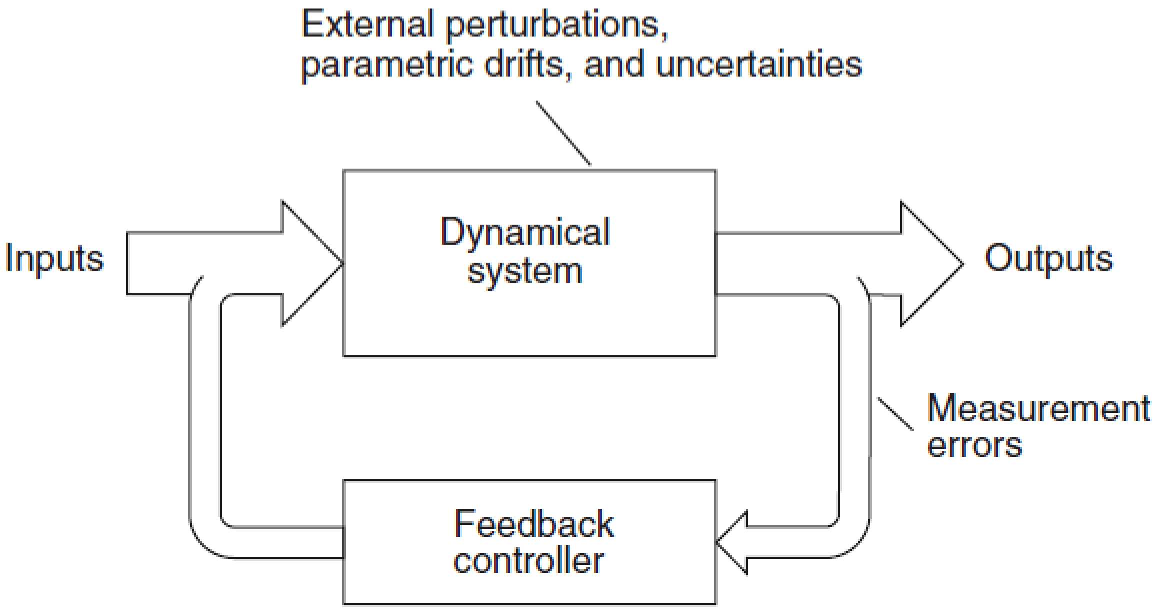
Source. Mabuchi et al. 2002



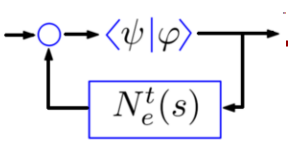
Control Problems In General

- Objective: Enhance system performance, despite noise and parameter uncertainty

- Process (Noisy)
- Control inputs
- Disturbances
- Controller
- Actuators



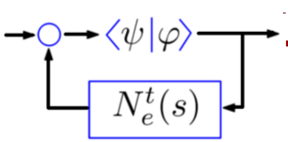
- Open Loop
  - No measurement
- Closed Loop
  - Measurement based feedback
  - Measurement noise



Open Loop Control In Quantum Systems

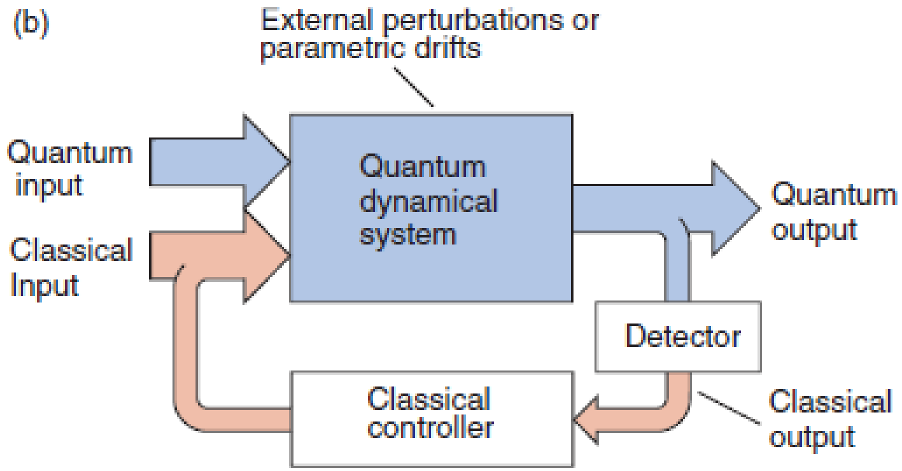
- Process time evolution (model)
  - Schrodinger Equation for wave function
  - Lioville Equation for density matrix
  - More complicated dynamics of open systems

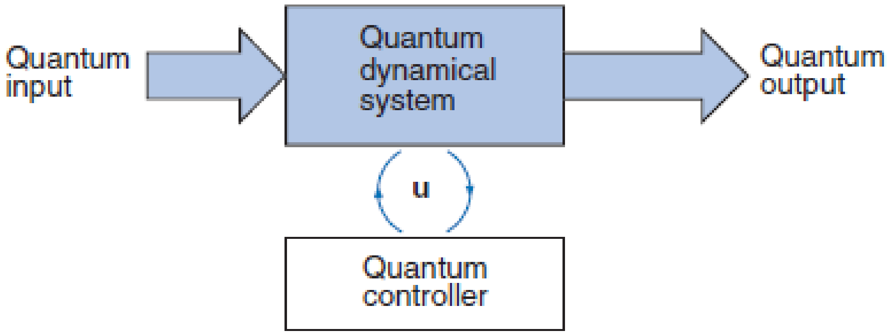
- Open Loop control
  - Design time evolution for an initial state to target a final state
- Closed Loop
  - Measurement based feedback
  - Measurement noise



Types of Quantum Feedback Control

- Measurements in the quantum world can be tricky (not found in classical control)
- Direct or indirect coupling between control and process (not found in classical control)





Information theory and Quantum Dynamics

- Evolution of a quantum system Q (and environment E)

$$\rho = U_{QE}\left(\rho_0 \otimes \rho^E\right) U_{QE}^\dagger$$

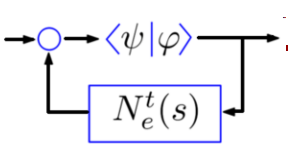
$$\rho^Q = \mathcal{E}\left(\rho_0\right) = \sum_i E_i \rho_0 E_i^\dagger$$

$$\text{von Neumann entropy } S = \operatorname{Tr} \rho^Q \log \rho^Q$$

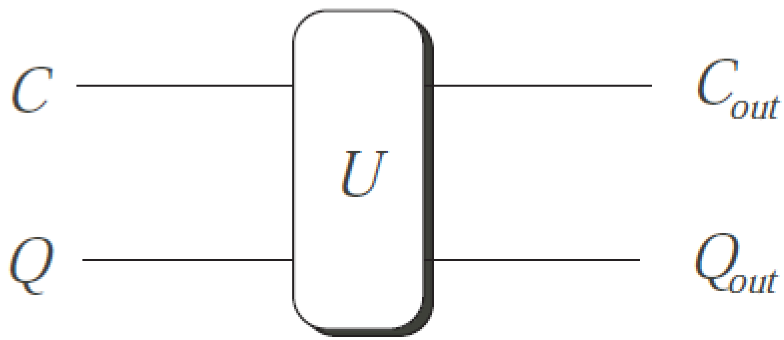
- Start with a pure state --- end up in a mixed one.
- The von Neumann entropy always increases.

$$S(Q) \equiv S(\rho^Q) > S(\rho_0)$$

- Question: Can you force the entropy to reduce?



Information theory and Quantum Control



Open Loop Strategy

Global Unitary operation

$$\rho^C = \sum_i p_i |i\rangle_C \langle i|.$$

$$\rho^{QC} = \rho^Q \otimes \rho^C = \sum_{i,j} p_i \rho_j^Q \otimes |i\rangle_C \langle i|$$

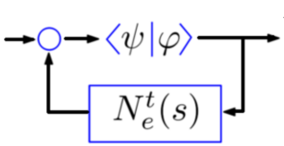
$$\rho^{QC} \rightarrow U_{\text{open}} \rho^{QC} U_{\text{open}}^\dagger$$

$$S(Q, C) = S(Q_{\text{out}}, C_{\text{out}}) \le S(Q_{\text{out}}) + S(C_{\text{out}})$$

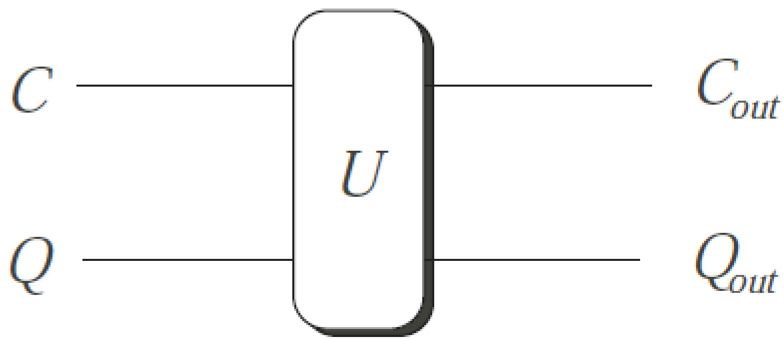
$$\Delta S_Q^{\text{open}} \equiv S(Q) - S(Q_{\text{out}}) \le S(C_{\text{out}}) - S(C)$$

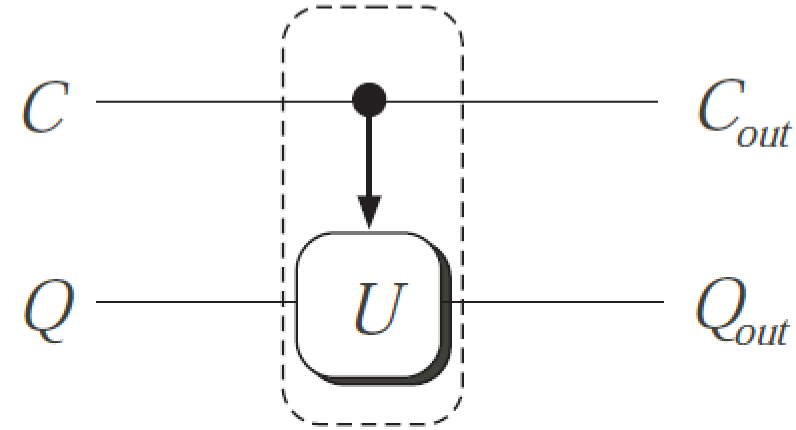
Entropy reduction.

Entropy reduction upper bounded by maximum amount of entropy increase of C.



Information theory and Quantum Control





$$U_{\text{open}} = \sum_i U_i \otimes |i\rangle_C \langle i|$$

$$\rho^{Q_{\text{out}} C_{\text{out}}} = \sum_{i,j} p_i U_i \rho_j^Q U_i^\dagger \otimes |i\rangle_C \langle i|$$

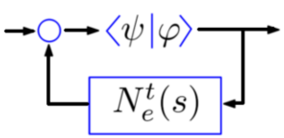
$$S(Q_{\text{out}}) = S\left(\sum_i p_i U_i \rho^Q U_i^\dagger\right)$$
$$\geq \sum_i p_i S\left(U_i \rho^Q U_i^\dagger\right) = S\left(Q\right)$$

Try another strategy

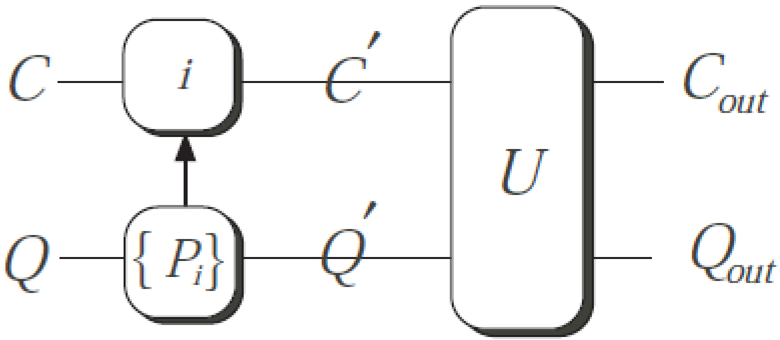
LOCC (Local Quantum operation, Classical Communication)

One can never reduce entropy this way

$$\Delta S_Q^{\text{open}} \leq 0$$



Information theory and Quantum Control



Closed Loop Strategy

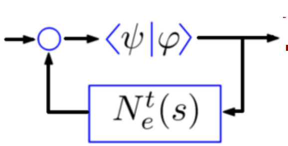
Use measurement that does not change entropy.

e.g. conventional von-Neumann

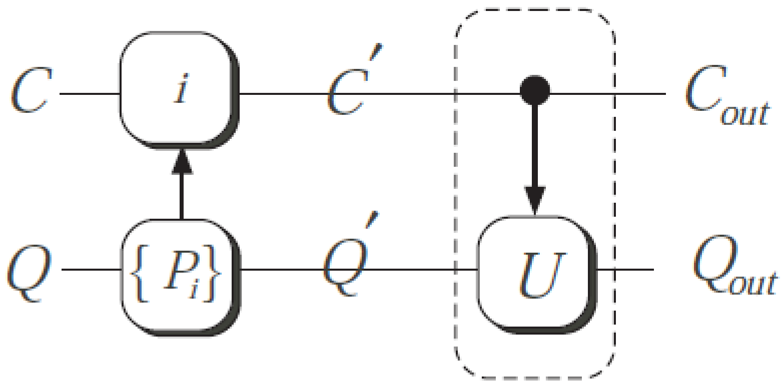
$$
\begin{aligned}
S(C_{\rm out}) &= S(Q_{\rm out}, C_{\rm out}) - S(Q_{\rm out}) + I(Q_{\rm out} : C_{\rm out}) \\
&= S(Q', C') - S(Q_{\rm out}) + I(Q_{\rm out} : C_{\rm out}) \\
&\leq S(Q) - S(Q_{\rm out}) + S(C') - I(Q' : C') \\
&\quad + I(Q_{\rm out} : C_{\rm out}), \qquad (13)
\end{aligned}
$$

$$
\begin{aligned}
\Delta S_Q^{\rm feedback} &= S(Q) - S(Q_{\rm out}) \\
&\leq S(Q', C') - S(Q_{\rm out}) + I(Q_{\rm out} : C_{\rm out}) \\
&= S(C_{\rm out}) - S(C') - I(Q_{\rm out} : C_{\rm out}) \\
&\quad + I(Q' : C') \\
&\leq \max_U \Delta S_Q^{\rm open} + I(Q' : C'). \qquad (14)
\end{aligned}
$$

Maximum improvement over open loop limited by quantum mutual info obtained by controller !



Information theory and Quantum Control



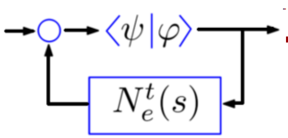
$$\Delta S_Q^{\text{feedback}} \leq I(Q' : C') = H(r_i)$$

$$r_i = \mathrm{Tr}(U_i P_i \rho^Q P_i^\dagger U_i^\dagger)$$

Closed Loop Strategy 2: LOCC

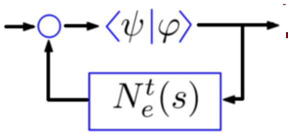
Maximum entropy reduction exactly equal to quantum mutual info obtained by controller.

This can be thought of as a quantum error correction process.



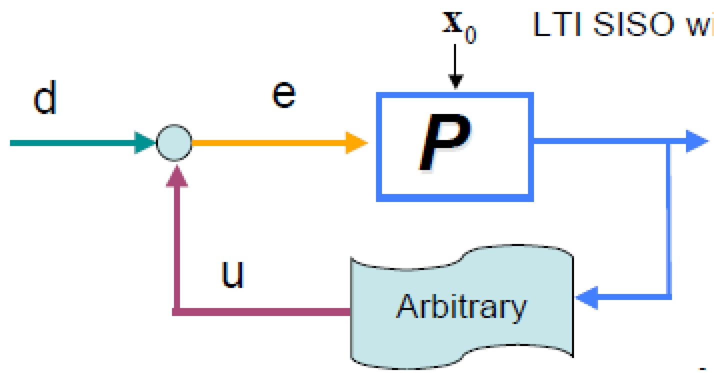
Where do we go from here?

- So far we have discussed feedback of classical information.
- What about coherent quantum feedback ?
- What if there is a limit on how much quantum information can be pumped back?
- Need to know about quantum capacity of quantum channels.



Role of Information in Control

LTI SISO with random initial state



$$S_{e,d}^2(\omega) = \frac{F_e(\omega)}{F_d(\omega)}$$

$$\frac{1}{2\pi}\int_{-\pi}^{\pi} \log S_{e,d}^2(\omega)\, d\omega \geq 2 \sum_{\text{unstable poles}} \log |pole_i|$$

Bode Integral Formula

$$h(\mathbf{e}^k) \geq h(\mathbf{d}^k) + I(\mathbf{x}_0; \mathbf{e}^k)$$

$$h_\infty(\mathbf{e}) \geq h_\infty(\mathbf{d}) + \liminf_{k\to\infty} \frac{I(\mathbf{x}_0; \mathbf{e}^k)}{k}$$

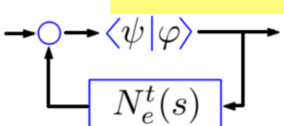
$$h_\infty(\mathbf{e}) \geq h_\infty(\mathbf{d}) + \sum_{\text{unstable poles}} \log |pole_i|$$

Stability implies:

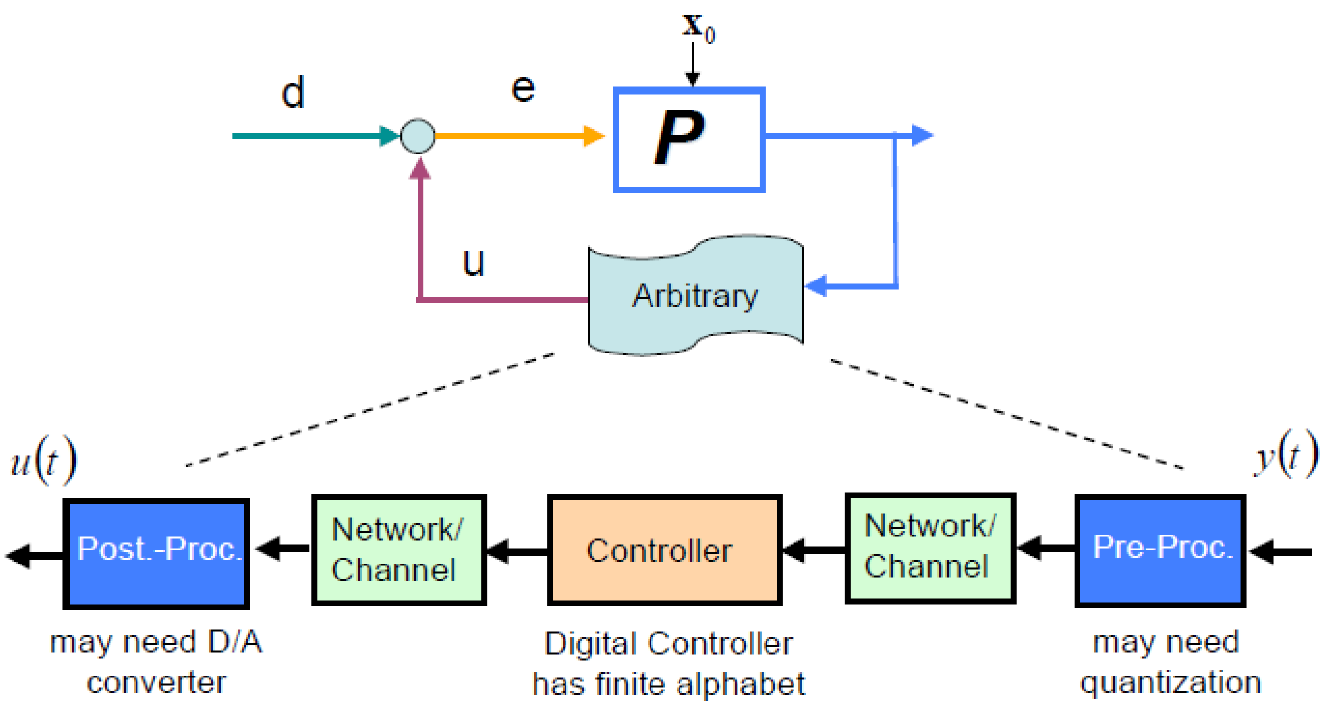
$$\liminf_{k\to\infty} \frac{I(\mathbf{x}_0; \mathbf{e}^k)}{k} \geq \sum_{\text{unstable poles}} \log |pole_i|$$

$$H(X) = -\sum_{x\in\mathcal{X}} p_X(x) \log p_X(x)$$

Classical Entropy

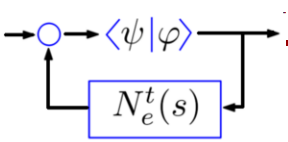


Role of Information in Control



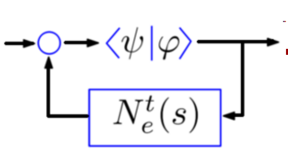
$C_f < \infty$ Finite Capacity Feedback

$$\frac{1}{2\pi}\int_{-\pi}^{\pi}\min\{\log(S_{e,d}(\omega)),0\}d\omega \geq \sum_{unstable\ poles}\log|pole_i| - C_f$$



What type of capacity?

- **No one knows the answer. (to-date)**
- **Capacity in feedback path is different from classical Shannon capacity of comm. channels.**
  - Anytime capacity (noisy channels) [Mitter et al.]
  - Topological feedback capacity (noiseless channels) [Nair et al.]
- **Open problem. What is the capacity definition to define feedback stabilization in quantum systems?**



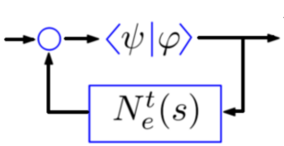
Why do we care about capacities?

- Communication engineer
  - Noise is ubiquitous
  - What are the physical limits of pumping info in communication channels in the presence of noise?
  - Does the quantum world offer any new limits/surprises?
  - Do quantum limits degrade communication capabilities?
- Experimental physicist
  - How sensitive can an apparatus be made?
  - Resolution: a parameter in the Hamiltonian settles to how many final states?
- Control engineer
  - How much information needs to be fedback in order to stabilize? (Quantum control)

Conclusions

- Quantum world has peculiar control structures
- Many problems in quantum information can be understood from control perspective
- The control perspective has a nice information theoretic explanation
- There are several open problems and avenues of research in this (young) area.
- Opportunity for engineers, physicists, computer scientists alike.

- **Thank you!**



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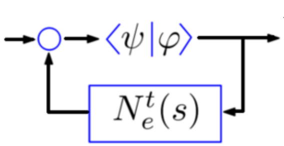

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