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WIT Summer School
May 23, 2017

Optimal Control:

AN APPLICATION TO GROUNDWATER MANAGEMENT

Natural Resource Economics

- Natural resources are scarce commodities
- We need to allocate scarce resources to make societies better off
- We also need to ensure that enough resources are left for future generations
- Economists strive to find the best way to allocate resources to make people better off today AND tomorrow

Marginal Willingness to Pay

- Implicit values attached to goods and services
- Marginal Willingness to Pay (MWTP):
  - The value placed on an additional unit of a good or a service
  - Downward sloping
  - Law of Decreasing Marginal Utility
- Also known as the inverse DEMAND function!
- Absence of markets to measure MWTP for environmental goods and services
- **What do we do?!**

Revealed Preferences

Hedonics

- Examine land markets
- Prices will reflect the existence of environmental goods and services

Travel Cost Method

- Examine people's expenditure on travelling to see (consume) environmental goods and services
- Examples: Baltoro Glacier; Keenjhar Lake; Indus Delta

Stated Preferences

Contingent Valuation

- Ask people directly what they are willing to pay for additional units of goods and services
- Surveys and questionnaires
- Captures both USE and NON-USE values
- Examples: Indus Blind Dolphin; Indus Delta Mangroves; Houbara Bustards

Groundwater

- Groundwater is a scarce renewable resource

- How should we allocate groundwater?
  - We can extract all of it today
  - But then there will be nothing left for future generations!

- We need to allocate groundwater across time periods (dynamic allocation)

Marginal User Cost of Groundwater

- The discounted value of the loss in future net benefits of a unit of extraction of groundwater in the present
  - Scarcity value
- Price per unit of groundwater includes not only its marginal cost of extraction but also its marginal user cost

Groundwater Management Regimes

Open Access Resource

- Myopic farmers (not forward looking)
- No incentives to conserve
- Resource degradation

Groundwater Management Regimes

Optimal Management

- Forward looking social planner
- Maximizes discounted (net present value) of net benefits of groundwater in a long term horizon
- Accounts for the marginal user cost of groundwater

Let's model farmer behavior under each of these management regimes!

Hydrological-Economic Model

Marginal Willingness to Pay for Groundwater:

$$W_t = g - kP_t$$

Where:

W_t is the units of groundwater extracted

P_t is the price per unit of groundwater extracted

g and k are parameters

Hydrological-Economic Model

Marginal Cost of Groundwater Extraction:

$$P_t = C_1 h_t$$

Where:

$C_1 h_t$ is the energy cost of pumping a unit of groundwater from a depth of h units

P_t is the price per unit of groundwater extracted

Evolution of the Stock of Groundwater

$$\dot{h} = \frac{R + (\alpha - 1)W_t}{As}$$

Where:

R is the natural recharge

α is the return flow coefficient

A is the area of the aquifer

s is the storativity coefficient

Open Access Behavior

Maximize Net Benefits in each period given the current value of the state variable:

$$\frac{g}{k} - \frac{W_t}{k} = C_1 h_t \qquad (1)$$

While

$$\dot{h} = \frac{R + (\alpha - 1)W_t}{As} \qquad (2)$$

$$W_t \geq 0,\ h_t = h_0 \text{ when } t = 0$$

Open Access Behavior

Equations (1) and (2) together define a ***first order linear differential equation***

Use an ***integrating factor*** to solve the differential equation

We will get solutions for h_t and W_t in terms of t and the exogenous model parameters

Draw Phase Diagrams to evaluate dynamics

Exponential convergence

Optimal Control

Maximize the present value of Net Benefits given the current and future states of the stock of groundwater:

$$Max \int_0^{\infty} e^{-rt} \left[\left(\frac{g}{k} W_t - \frac{1}{2k} W_t^2 \right) - (C_1 h_t W_t) \right] dt$$

Subject to:

$$\dot{h} = \frac{R + (\alpha - 1) W_t}{As}$$

$$W_t \geq 0, h_t = h_0 \text{ when } t = 0$$

Optimal Control

The present value Hamiltonian is given by:

$$\mathcal{H} = e^{-rt}\left[\left(\frac{g}{k} W_t - \frac{1}{2k} W_t^2 \right) - (C_1 h_t W_t)\right] + \lambda_t \left(\frac{R+(\alpha-1)W_t}{As}\right)$$

Necessary Conditions:

$$\frac{\partial \mathcal{H}(W_t, h_t, \lambda_t)}{\partial W_t} = 0$$

$$\dot{\lambda}_t = -\frac{\partial \mathcal{H}(W_t, h_t, \lambda_t)}{\partial h_t}$$

$$\lim_{t \to \infty} \lambda_t\, h_t = 0$$

Optimal Control

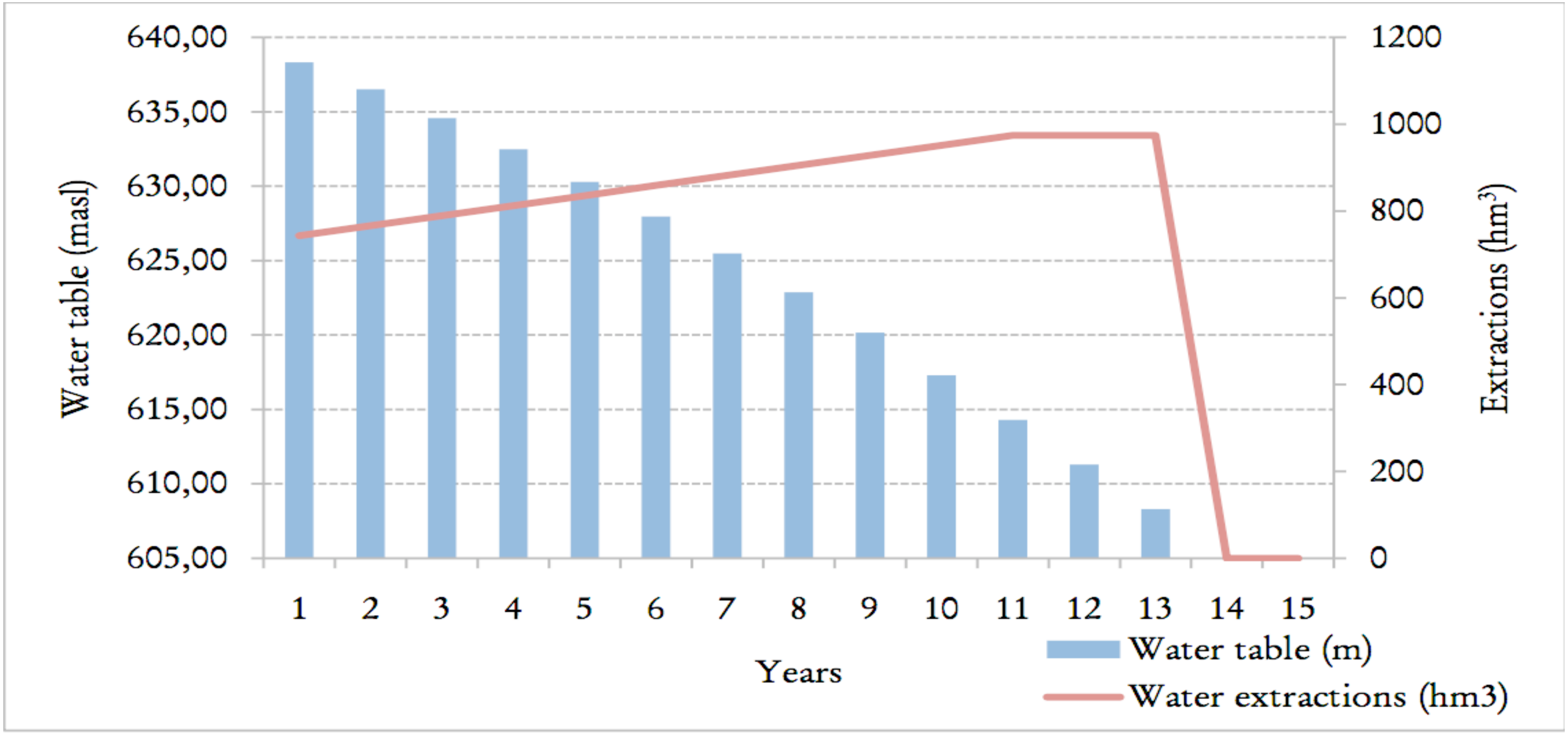
System of “complex” differential equations

Can be further solved to get solutions for h_t and W_t in terms of t and the exogenous model parameters

Draw Phase Diagrams to evaluate dynamics

Exponential convergence

Example: Encarna and Albiac 2010



Example: Encarna and Albiac 2010