RESEARCH STATEMENT

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Summary. I do fundamental research at the interface of computational geometry, distributed algorithms, communication networks and control theory, concerning three prominent yet overlapping areas of research. First, I am investigating complex networked systems that will enable the future deployment of very large-scale and ubiquitous instances of sensor networks, robotic swarms and mobile networking. Second, I study massive high-dimensional data sets in various settings for rapid information discovery. Third, I am exploring quantum information theory and quantum control to understand better the physics of information. My approach in all three directions shares an emphasis on global topological methods, modes of information propagation and the role of feedback.

Connections & Complexity: Large-Scale Distributed Networks and their Dynamics

Perhaps, the best word that summarizes the behavior of today's global society is *connectivity* [1]. The pervasive influence of networks in all aspects of life - biological, physical, and social - has led researchers on the quest to discover fundamental principles underpinning complex networked systems. Network phenomena manifest themselves in all aspects of biological diversities, from biochemical reactions and neural networks to insect swarms and complex ecological systems [2]. These effects are also visible in fundamental physics, for example in lattices ensembles of particles in statistical physics and spin networks in most variants of quantum gravity. They are also found in such diverse engineered systems as power grids, communications networks, irrigation networks and transportation infrastructures. Connectivity is also visible in all sorts of human social interactions such as collaboration, coordination and exchange of information, thus enabling the necessities and conveniences of modern life. All such network phenomena share a common principle — the emergence of *global behavior* from *local interactions* via adaptation and cooperation. Thus systematic studies of these remarkably diverse and pervasive examples hold great promises for solving numerous engineering and scientific problems. Research on *networked sensing & control systems* has already enabled the design, analysis and deployment of new types of networks, from *sensory* to *robotic* and from *wireless* to *adhoc*.

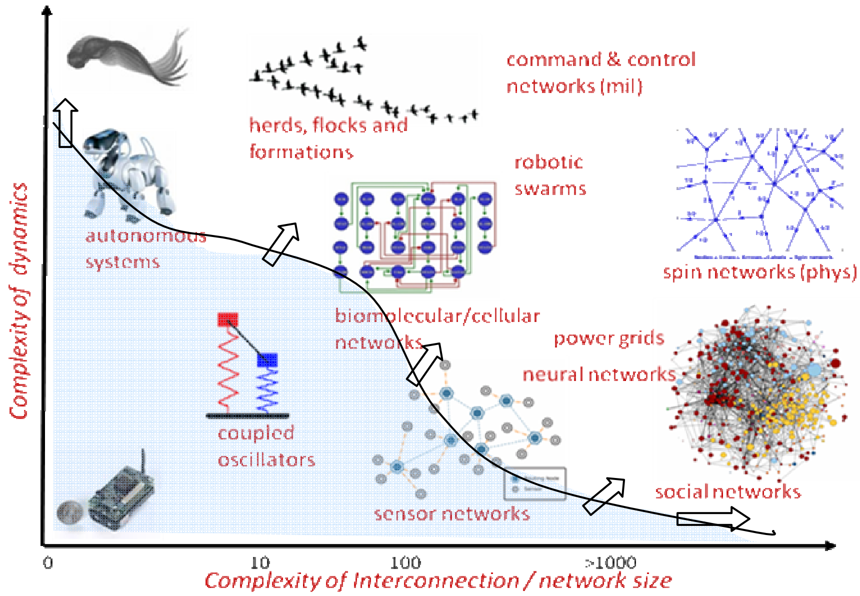


Figure 1. The research frontier in complex networked systems research. Examples of these systems are marked on a graph in which the dynamical complexity of individual agents is plotted against the complexity of interconnection. These examples span a diverse and pervasive range of natural and engineered systems, and carry high scientific value and social relevance.

In Figure 1, two prominent aspects of the complexity of complex networked systems have been depicted. Researchers in autonomous system have recently made many landmark achievements that reflect the high-level of expertise in dealing with the complexity of individual system dynamics. In 2005, a driverless vehicle autonomously covered 130 miles of desert territory in less than seven hours to win the DARPA grand challenge on robotics [13]. By the same year, tactical- and theater-level unmanned aerial vehicles had flown over 100,000 flight hours in support of combat missions around the globe. On the other hand much of the current work on large-scale networks assumes that individual agents have very simple or no dynamics. It is therefore time to push the frontiers of this research and envision systems made up of hundreds or perhaps thousands of reliable robotic platforms, linked by reliable communication networks for applications hitherto unthinkable [3]. However, the realization of such distributed dynamical systems is hindered by several challenges.

It is worth noting that the main challenge in this field has shifted from difficulties in manufacturing to the lack of theoretical foundations for provably correct design and deployment. There are many gaps in the fundamental understanding of collective dynamics of networks. There is a need to develop tools, abstractions, and approximations that provide a rigorous mathematical basis in order to explain common concepts across fields; model uncertainties characterized by noisy and incomplete data; incorporate experimental measurements; and find systems that are both robust and secure [1], [2]. Many of these issues are *structural*: how does one characterize the wiring diagram of a metabolic network or a food web or the Internet or the visual cortex? Are there any unifying principles underlying their *topology*?

Another important question is whether there are common laws that explain the modes of *information propagation* in diverse networks such as insect colonies, the world-wide web, electrical power grids, citation networks of scientists etc? If so, can these principles be used to construct optimal coordination strategies for various applications of networked sensing & control? One of the most important issues in studying information propagation is the role of *feedback* (Figure 2). How do networks self-stabilize globally using only local information? What is the minimum information that needs to be exchanged among agents to reliably close the loop over noisy communication channels while obeying bandwidth requirements? Studies on the effects and limitations of feedback in networks are of critical importance to congestion control, network protocols and cross-layer optimization for traditional networks. They are also leading to new applications such as remote robotic surgery and deep-space mission control. Needless to say, that these studies have direct relevance to the many outstanding scientific problems mentioned above. The two issues, namely the topological structure of distributed systems and information feedback & propagation are the focal points of my research on complex networks. In both questions, one needs to understand the complex interplay between the four basic ingredients of a dynamic network: *sensing, communication, computation,* and *control.*

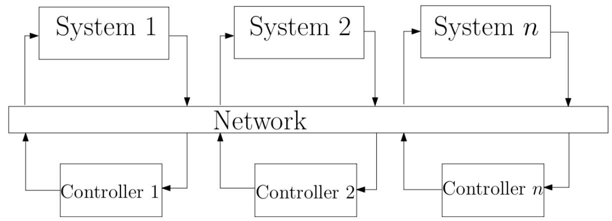
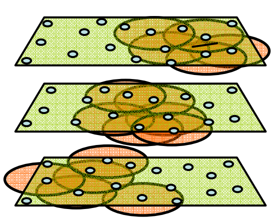
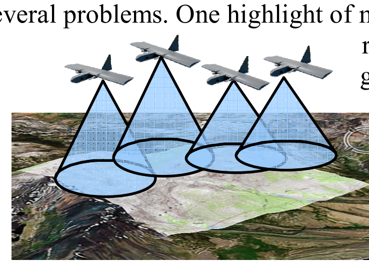


Figure 2. Information propagation, feedback and control in distributed networked systems.

As the size and complexity of such systems increase, the spatial interactions between subsystems become intractable for design and analysis by traditional methods alone. One highlight of my research has been to capture such seemingly intractable complexities by combining traditional techniques of systems theory with novel abstractions from graph theory and algebraic topology [4]. These abstractions are concise, robust and provably correct representations of the redundant geometric information in the system. They are strongly motivated by the physical characteristics of networks, such as sensory and communication constraints. Moreover, they allow a natural gluing of local information into global network characteristics. Although there is a wealth of work on computational applications of graph theory and algebraic topology, the unique characteristics of networked control systems has led me to think about new methods and algorithms that are well suited for actual realization. These methods are *distributed* and cognizant of network limitations such as information routing constraints, bandwidth and energy.

In my doctoral thesis and postdoctoral work, I have succeeded in demonstrating the promise of this research philosophy for several problems. One highlight of my doctoral thesis is a motion planner for dynamic reconfigurable networks that uses a combination of graph embedding techniques, semi-definite programming and standard motion planning. In another study, I solved the blanket coverage problem for sensor networks using a novel application of computational algebraic topology, resulting in several degrees of robustness and simplification over standard algorithms. In my postdoctoral work, I have generalized these results to the dynamic coverage scenario (See figure). This has been done by combining ideas from hybrid control systems, discrete-differential geometry and harmonic analysis to get a distributed verification algorithm for *all* dynamic coverage algorithms. In another work, I have proposed some decentralized algorithms for computing certain topological invariants. These invariants have been shown to be related to network characteristics such as information routing in communication networks, coverage gaps in sensor networks and coordination mechanisms in multi-agent robotics. In short, all these studies support the hypothesis that a common language to describe many types of network phenomenon can be obtained by the correct mathematical abstractions that focus on two most fundamental aspects of networked systems, namely distributed information propagation and topology characterization.





Topology of Data: Exploiting Minimal Geometry for Rapid Information Discovery

The increasing pervasiveness & accuracy of sensors, unprecedented automation of data collection, extremely cheap storage and rapid dissemination by communication networks have resulted in an explosion of data to deal with. In particular, advances in imaging and scanning techniques have resulted in massive data sets in several fields, from genome sequences, medical images and satellite imagery to multimedia, scans of molecular surfaces and environmental recordings [5]. Typical examples of such databases are the sets of measurements sampled from spatial structures, also known as *point cloud data* (See Figure 4). Examples of point cloud data extend from traditional probes of objects in 3D to a wide variety of high-dimensional sets generated by neuronal activity, network evolution, astronomical data, robotic sensing etc.

Hidden in these high-volume databases and data flows are some highly *nonlinear* properties and geometric structures that are easier to describe qualitatively than by quantitative methods. Moreover, even though these data sets are *high-dimensional,* they typically reside on sets of much lower dimension and non-trivial *topology*. Traditional statistical methods, data-mining and machine-learning techniques are unable to decipher this information due to the *curse of dimensionality*, linearity assumptions and non-incorporation of global topological information. Therefore, new and efficient computational techniques that employ *dimensionality reduction* and *topology discovery* hold great promises for resolving outstanding scientific problems and generating new applications by rapid information discovery [4], [5]. These techniques can be grouped broadly under the subject of *computational topology*.

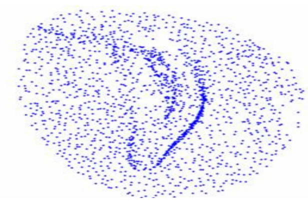


Figure 4. A point-cloud data set in 3D. A wealth of information is hidden in high dimensional analogs of such sets that can be discovered by topological methods.

My interest in this subject was originally motivated by the sets of data generated by complex networked systems described in the previous section. My thesis was one of the earliest research works on networked sensing using this approach. I am especially interested in pushing the research frontier (cf. Figure 1) in networked systems in all directions of complexity by asking the following question. Can complicated nonlinear dynamics, whether individual or collective, be described more effectively and robustly by global qualitative measures? Can classical notions of topological and *symbolic dynamics* be augmented with *computable* descriptions of feedback, stability, uncertainty and most importantly, undetermined input or *control*. Another source of interest for me is point-cloud data sets in sensory data generated in robotics [4]. Can massive measurement sets from laser-scans, point probes or sonar echoes be assembled, parsed and abstracted for motion planning, environmental modeling, localization and mapping etc?

Topological techniques for information discovery share two prominent characteristics: the inference of high dimensional properties from low dimensional projections and secondly, the emergence of global continuous information by assembling discrete local properties [6]. In my postdoctoral work, I have already explored both these questions for particular settings. For example, how can certain operators on combinatorial spaces be refined to give an analogous meaning for the smooth setting? How do we interpret and visualize ‘flows’ on discrete spaces in a fashion similar to continuous spaces? And most importantly, how do these approximations obey global topological obstructions? While such questions are sometimes answerable for particular situations, generalizing these results is an important research direction towards realizing discrete versions of differentiable calculus, geometry and mechanics. Such discrete and *computable* calculi have been recognized by researchers in many research areas as important for future applications of electromagnetics, fluid dynamics, computer graphics, data visualization and computer vision.

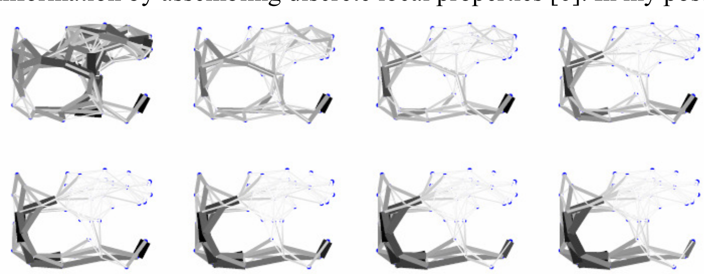


Figure 5. A discrete flow over a combinatorial space, inspired by a geometric diffusion process. Such flows have been used in my postdoctoral work to discover underlying topological invariants in network data.

Physics of Information: Communication, Control and Computation at the Quantum Scale

An important scientific question today is to understand the fundamental limits of information exchange across scales. As devices are being manufactured at smaller and smaller scales, *quantum effects* are expected to play a prominent role in their design and analysis. These effects are commonly thought of as obstacles or limitations in the continuation of current trends in microelectronics and nanotechnology. However, physicists, computer scientists and mathematicians have recently realized how to exploit these effects to their advantage in future devices and applications [7]. The field of *quantum information sciences* [8] encompasses many fundamental questions which have excited my interest and I have begun an exploration of these questions in my current postdoctoral work. In particular, I am interested in two broad themes in quantum information sciences: control of quantum mechanical systems and quantum information theory.

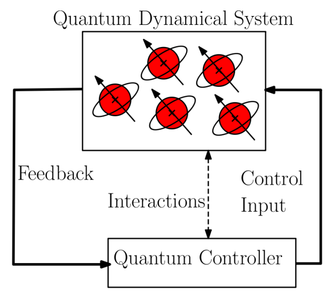


Figure 6. Quantum control.

Quantum control is an emerging technology, driven by investigations into quantum computation, nanotechnology, material synthesis, laser physics and NMR (nuclear magnetic resonance). It concerns the control of those physical systems in which the dominant behavior is quantum mechanical. From practical realizations of quantum computers to error compensation in NMR systems, it is vitally important to understanding all aspects of controlled quantum dynamics. Feedback control not only enables trajectory design for the desired evolution of the system, but is also invaluable for compensation of systematic errors, cancellation of disturbance and suppression of noise. My interest in quantum control concerns the design of pulse sequences for accurate NMR control and quantum gate approximation. The basic idea is to assemble a string of operations chosen from a small set of experimentally realizable primitives to produce arbitrary dynamics.

My other interest in quantum information sciences concerns *quantum information theory.* This is a relatively young field whose aim is to generalize Shannon's pioneering work in classical information theory to the quantum setting. Explorations in this area have promised schemes for teleportation, secure communication and cryptography. As with classical information, the main object of study is a (quantum) communication channel over which information is transported in *qubits* – the quantum analog of bits. The capacities of such channels; coding, compression & modulation schemes and communication security are main themes in this area. Quantum modes of communication, augmented with classical communication schemes give rise to *quantum networks.* Currently, my interest in quantum information theory concerns the following question: Is there an information theoretic characterization of feedback in quantum controllers and quantum networks? Since this problem has already been studied in recent networked control literature, it is my hope that by re-examining these notions in the quantum setting, several fundamental properties of quantum controllers and quantum networks can be derived.

Logistics: Towards establishing a world-class research group

Short-term goals

Executing the research program proposed in the previous sections will critically depend on establishing the following laboratories and research teams over the period of 3-5 years.

- *Networked Dynamical Systems Laboratory (NDSL):* A research lab in networked sensing and control; comprising of 3-4 graduate students in electrical engineering or computer science, 1 postdoctoral researcher and 2 lab engineers. The laboratory will consist of the following equipments and setups.
  - *Sensor network test-bed:* A network of 100+ nodes using standard mote technology.
  - *Networked robotics test-bed:* A test-bed for networked distributed control composed of 8-12 programmable robotic ground vehicles, 4-5 autonomous aerial vehicles, advanced sensors and communication equipment.
- *Theoretical research Group:* A research group in applied mathematics; focusing on the computational applications of topology, complex systems, control theory and quantum information sciences. The group will comprise of 2-3 graduate students in mathematics or computer science and 1 postdoctoral researcher.
- *Quantum Information and Control Laboratory (QICL):* A research lab to investigate feedback control and communication phenomenon in quantum mechanical systems using experiments in Nuclear Magnetic Resonance (NMR) and/or quantum optics. The laboratory will be staffed by 1-2 graduate students and 1 postdoctoral researcher in physics or electrical engineering.

Budget and Resources

To initiate these research activities, guaranteed financial support has to be provided by the employer for the first 3 years of employment. Various public and private funding agencies will also be requested in due time to support this research. The total funds should be sufficient to support myself, along with 6-8 graduate students, 2 postdoctoral researchers and 2 lab engineers over a period of 5 years. These funds should also include compensations for travel and international collaborations.

In addition to salaries, funds will be needed to setup two laboratories. NDSL will be setup in the first two years of appointment, requiring an annual budget of $250,000 for equipment without including overheads for laboratory space and maintenance. QICL will require a budget of $500,000 with similar constraints. It is expected that these costs can be reduced significantly, if (a) some of the required equipment is manufactured indigenously within the laboratory, and (b) the resources are shared with other faculty with similar research interests.

Long-term goals

The goals and milestones mentioned above will help achieve the following long term goals.

- Establish world-class interdisciplinary research programs for fundamental research in complex networked systems and quantum information sciences.
- Induce industrial, commercial and social applications of the above-mentioned fundamental research via improvement of urban transportation, irrigation networks and communication infrastructures; decentralization in industrial management and government policy & planning; automation of data collection and information security; exploration of surveillance mechanisms and related privacy issues [7].
- Develop relevant technologies for scientific problems of high importance such as environmental studies, wildlife habitat monitoring, natural resource conservation, geological survey etc.
- Explore the implications of this research for other areas in mathematical sciences [6].
- Establish collaborations across disciplines, especially with mathematicians, physicists, computer scientists, life-scientists and engineers.

Unification: How does this all fit together?

As pointed out in the summary, my three main research areas share many common themes. At a more fundamental level, there is a pivotal reliance on the power of abstraction to deal with similar problems in different settings using a single unified language – the language of mathematics. For example, by considering the pathways in a gene regulation circuit and coordination schemes in a team of robots as instances of the same abstract problem, one can not only provide insights into both of these problems but also solve the same problem for other physical instantiations of that abstraction. This is the same process of *unification*, that led Newton to model revolving planets and falling apples using the same equations of motion, and the grand unification that scientists sought to this day [9].

Keeping in mind that the ignorance of experimental science becomes a hurdle in creating good theory, the real challenge in applied research amounts to recognizing the critical points where experimental data cannot be explained without unifications. But hundreds of years of research in the mathematical sciences have taught us how to find *good* unifications [10]. The abstract unified picture should provide a surprise – something that the disjoint perspectives cannot reveal independently. It must generate new insights and provide correct predictions that cannot be obtained without resorting to the abstraction. For problems in applied research, one should also add efficient computability. I believe that all three areas in my research program are excellent candidates for such unifications.

Moreover, all three promise *scientific revolutions* in the sense of Kuhn [11]. They are banked on dramatic shifts from existing methods and paradigms; respond positively to the challenges unanswered by current science; and finally promise more elegant and simpler solutions to already solved problems. The proposed research has strong connections to both the greatest engineering challenges that we face today [7] as well as the most important unresolved mathematical mysteries of our time [6]. The recent resolution of the Poincaré conjecture in mathematics [6], [12] and the breathtaking performance of autonomous robotic vehicles in the recently held DARPA grand challenges on robotics [13] are heartening assurances that a research program banking on the maturity of abstract techniques in topology and the reliability of engineering design in robotics has a great chance of success.

Finally, I feel great satisfaction in the fact that my training in both engineering and mathematics has allowed me to appreciate the subtle interplay between technology, science and society from a historical and philosophical perspective. In particular, the rise and fall of various medieval and pre-modern scientific traditions, especially of the Islamic and Indian civilizations are of great interest to me. Such historical perspectives are critical for understanding the social and ethical implications of new technologies. The 'brave new world' offered by technology alone can not be a solution to the problems facing mankind unless it pays due attention to all aspects of the mind, body and soul.

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