WHITING SCHOOL OF ENGINEERING
JOHNS HOPKINS UNIVERSITY

520.435 Digital Signal Processing

Hassan Mohy-ud-Din

Course Page : http://cyphynets.lums.edu.pk/index.php/ECE_520.435_-_DSP_for_Matlab
My Webpage: http://cyphynets.lums.edu.pk/index.php/Hassan_Mohy-ud-Din

Z Transform

$$X(z)=\sum_{n=\infty}^{\infty} x[n]z^{-n}$$

$$x[n]=\frac{1}{2\pi j}\oint_C X(z)z^{n-1}dz$$

$$=\sum[\text{Residues of } X(z)\cdot z^{n-1} \text{ at the poles inside the contour } C]$$

Pole-Zero Plot

$$H(z) = K\frac{(z-z_1)(z-z_2)\cdots(z-z_n)}{(z-p_1)(z-p_2)\cdots(z-p_n)}$$

Transfer Function Polynomial Coefficients

$$H(z) = K\frac{Num(z)}{Den(z)}$$

$$G(z) = \frac{1-24z^{-1}+288z^{-2}}{1-08z^{-1}+064z^{-2}}$$

[Z,P,K]=tf2zpk(Num, Den)

Discrete-time Transfer Function to Zero-Pole Conversion

Plotting in Z-Plane

zplane(z,p)

Plots the zeros and poles with the Unit circle as reference

z-row vector of zeros of H(z) or roots of Num

p-row vectors of columns of H(z) or roots of Den

Digital Filter Frequency Response

$$H(e^{jw}) = \frac{B(e^{jw})}{A(e^{jw})} = \frac{b(1) + b(2)e^{-jw} + \cdots + b(m+1)e^{-jmw}}{a(1) + a(2)e^{-jw} + \cdots + a(n+1)e^{-jnw}}$$

[H,w] = freqz(Num,Den,N)

H: N-point complex frequency response vector
W: N-point frequency vector, w, in radians/sample
N: resolution of frequency response

Z-Transform Partial Fraction

$$\frac{B(z)}{A(z)} = \frac{r(1)}{1-p_1 z^{-1}} + \cdots + \frac{r(n)}{1-p_n z^{-1}} + K_1 + K_2 z^{-1} \cdots$$

$$number\ of\ \ poles, n = length(Den) - 1 = length(R) = length(P)$$

$$if\ \ length(Num) < length(Den), then\ K = [],$$

$$length(K) = length(Num) - length(Den) + 1$$

$$if\ \ P(j) = \cdots = P(j+m-1)\ is\ a\ \ pole\ of\ \ algebraic\ multiplicity, m, then:$$

$$\frac{R(j)}{1-p_j z^{-1}} + \frac{R(j+1)}{(1-p_j z^{-1})^2} + \cdots + \frac{R(j+m-1)}{(1-p_j z^{-1})^m}$$

Residue Computation

Reversible Function

[R,P,K] = residuez(Num,Den)

R: Residues of partial fraction expansion of H(z)

K: Direct Terms of partial fraction expansion of H(z)

P: Poles of H(z)

Residuez(.) *is a discrete equivalent for computing the partial fraction expansion of H(z) and we* ***do not*** *need to divide by* ***z*** *initially to obtain the coefficients.*

Residue(.) *is a continuous equivalent for computing the partial fraction expansion of H(z) and we* ***do*** *need to divide by* ***z*** *initially to obtain the coefficients*

Inverse Z-Transform

$$X(z) = \frac{1}{(1-0\cdot 9z^{-1})^2(1+0\cdot 9z^{-1})}, \ |z| > 0.9$$

Computing the Num and Den and then applying Residue Theorem.

Q. But MATLAB is supposed to reduce work, can I calculated the Den coefficient directly ?

A. Yes, you can. Using poly(.)

System Representation in Z-Domain

$$H(z) \triangleq Z[h(n)] = \sum_{n=-\infty}^{\infty} h(n) z^{-n}; \quad R_{h-} < |z| < R_{h+}$$

$$Y(z) = H(z) X(z); \quad ROC_y = ROC_h \cap ROC_z$$

$$y(n) + \sum_{k=1}^{N} a_k y(n-k) = \sum_{l=0}^{M} b_l x(n-l)$$

$$H(z) = b_o z^{n-m} \frac{\prod_{l=1}^{N} (z - z_l)}{\prod_{k=1}^{N} (z - p_k)}$$

Transfer Function Representation

$$H(e^{jw}) = b_o e^{j(N-M)w} \frac{\prod_{1}^{M} (e^{jw} - z_l)}{\prod_{1}^{N} (e^{jw} - p_k)}$$

$$|H(e^{jw})| = |b_o| \frac{|e^{jw} - z_1| \ldots |e^{jw} - z_M|}{|e^{jw} - p_1| \ldots |e^{jw} - p_N|}$$

$$\angle H(e^{jw}) = \underbrace{[0 \; or \; \pi]}_{\text{constant}} + \underbrace{[(N-M)w]}_{\text{linear}} + \underbrace{\sum_{1}^{M} \angle(e^{jw} - z_k) - \sum_{1}^{N} \angle(e^{jw} - p_k)}_{\text{non-linear}}$$

Solutions to the Difference Equation

One-sided Z-Transform of a sequence **x[n]** is:

$$Z^{+}[x(n)] \triangleq Z[x(n)u(n)] \triangleq X^{+}[z] = \sum_{n=0}^{\infty} x(n) z^{-n}$$

$$Z^{+}[x(n-k)] = x(-1)z^{1-k} + x(-2)z^{2-k} + \ldots + x(-k) + z^{-k} X^{+}(z)$$

The above equation can be used to solve the difference equation:

$$1 + \sum_{k=1}^{N} a_k y(n-k) = \sum_{m=0}^{M} b_m x(n-m), \quad n \geq 0$$

subject to the initial conditions:

$$\{y(i), i = -1, \ldots, -N\} \text{ and } \{x(i), i = -1, \ldots, -M\}$$

Defining Initial Conditions

filter(b, a, x, Zi)

Zi=filtic(Num, Den, Yi, Xi)

converts past inputs and outputs into initial conditions for the state variable Z