Algorithm for the Jordan Form of A

1. Compute the distinct eigenvalues $\lambda_1, \lambda_2, \ldots, \lambda_s$, along with the associated algebraic multiplicities $m_1, m_2, \ldots, m_s$ and geometric multiplicities $\mu_1, \mu_2, \ldots, \mu_s$.

2. Treat each eigenvalue in turn. For a given eigenvalue λ of algebraic multiplicity m and geometric multiplicity μ, we start computing the **E-spaces** and their dimensions. The k-th E-space is
$$E_\lambda^k = \left\{ \mathbf{x} : (A - \lambda I)^k \mathbf{x} = \mathbf{0} \right\}$$
So E_λ^1 is just E_λ, and we build from there. We stop when we get to an E_λ^k that has dimension m, the algebraic multiplicity of λ.

3. We make a diagram of boxes as follows. Compute the numbers
$$
\begin{aligned}
d_1 &= \dim E_\lambda^1, \\
d_2 &= \dim E_\lambda^2 - \dim E_\lambda^1, \\
&\vdots \qquad \vdots \\
d_k &= \dim E_\lambda^k - \dim E_\lambda^{k-1}.
\end{aligned}
$$
Now make a diagram with d_1 boxes in the first row, d_2 boxes in the second row, and so on. For example, if $d_1 = 4, d_2 = 2, d_3 = 2, d_4 = 1$, then we get a diagram

$$
\begin{array}{l}
\square\square\square\square \\
\square\square \\
\square\square \\
\square
\end{array}
$$

We are going to 'fill in' the diagram with vectors as follows.

4. Start at the bottom of the diagram, and fill the boxes in row k with linearly independent vectors that belong to E_λ^k **but not** E_λ^{k-1}. Anytime you have a vector v in a box, the box immediately above it gets filled with the vector $(A - \lambda I)v$. If a box is the lowest in its column, and belongs to row i, fill that box with a new vector from E_λ^i, which is linearly independent to both E_λ^{i-1} and all the other vectors in row i.

5. Repeat steps 2 through 4 for each distinct eigenvalue. You will get a diagram full of vectors for each one.

6. Make a matrix Q as follows. For each eigenvalue, consider the associated diagram. The vectors in the boxes become the columns of Q as follows. Start at the top of the leftmost column, and use the vectors as you go down the column. When you reach the end of a column, go to the next column. When you finish one diagram, go to first column of the next diagram. This gives the matrix Q.

7. The Jordan form of A is given by $J = Q^{-1}AQ$. But the nice part of the algorithm is that you can compute J **without finding** Q! In fact J will have one Jordan block for each column of each diagram. The value of the block is given by the eigenvalue, and the size of the block is equal to the number of squares in the column. You put the blocks down the diagonal of J in the same order you chose the vectors in Q.