Locating Leaks & Dumps in Open Channels with Minimal Sensing

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Abstract— **In this paper, we investigate methods for automated localization of leaks and dumps in an open channel flow. We propose a framework under which we solve this important system health monitoring problem by using a network of level sensors and estimation algorithms. We first discuss numerical techniques for simulating a channel with leaks or dumps. We then give results of a multiple-model estimation technique and study its performance against model uncertainty and noise. We find the method to be both robust and accurate. We also find that the sensing accuracy increases at least quadratically by addition of sensors. This is achieved by using a search algorithm on the errors generated by multiple PDE models.**

I. Introduction

One of the major causes related to the scarcity in distribution of water in many developing world countries is the inefficiency of their irrigation networks (For a global survey, see [16]). In countries like Pakistan, India and Egypt, the irrigation networks are extremely vast and complex, the water sources are few and drying, while the infrastructure is in a constant decay, dating back by decades and centuries ([4]). One major governance issue faced by such developing world irrigation canal infrastructures is the unregulated and unauthorized dumping of industrial wastes and sewerage into open channels meant for irrigation. There are obvious repercussions of this practice such as pollution, which can make water unsuitable for drinking or agriculture; blockage of water courses; and increase in flows which may lead to flooding. Similarly, there is a widespread practice of unauthorized diversion of canal water for irrigation or industrial use. This results in difficulties of pricing, decrease in water distribution efficiency, damage to control infrastructures and inequity amongst upper and lower riparian communities. Finally, there may be impulsive decreases in flows due to breaches caused by natural deterioration or impulsive increase in flows due to flash floods.

All of the above mentioned conditioned can be attributed to some form a localized *leak* or a *dump* into canal system which needs real-time rectification for a safe and efficient operation of the system. Unfortunately, early damage control requires early detection and localization which is not available in most developing world irrigation infrastructures due to the lack of instrumentation and automation as well as the geographic extent of the networks. To give an idea, the main canal network in Pakistan is $90,000$ km in length [4]. In an earlier paper [12], the authors have proposed a Cyber Physical Systems (CPS) architecture towards the modernization of the current infrastructure with state of the art methods in systems engineering and sensing technologies, primarily for regulation of water levels. In this paper, we propose that we can reuse the same infrastructure for detecting leaks and dumps in the system by adding minimal sensing requirements.

There exists a vast body of literature on modeling the flow of water in open channels ([6], [17]) and controlling water levels in an irrigation canal network using methods of feedback control ([9], [5], [15]). Various control theoretic problems have been addressed and solved by researchers related to modeling [7], controller design ([14]), decentralization ([8], [13], [5]), security ([1]) and practical implementation issues ([10], [14]).

We give below the various factors in choosing or developing particular techniques for leak detection.

1) Applicability to a very large-scale, geographically extent network.
2) Design and testing in a low-cost (preferably virtual) laboratory setting
3) Decentralization to allow flexibility in governance and scalability.
4) Fitness for unreliable energy and telecommunication infrastructures.
5) Minimal cost addition and modification to canal command infrastructure.

The paper is organized as follows. We first setup the problem in Section II and explain the computational and sensing infrastructure used in the methodology. In Section III, we develop and simulate dynamical models for open channel flows with a known geometry and control parameters. Next, we decouple the problem of leak/dump detection in time from the event's localization in space (Section IV). Using both the canal geometry and event start and stop time, we attack the localization problem in Section V. Next, we study the robustness of our method by simulating the effects of model uncertainty and by incorporating sensor noise (VI). Finally, we hint towards some possible extensions such as incorporation of prior history of events, multiple breaches etc. in Section VII.

II. Problem Formulation and Setup

Consider a pool of open channel flow, (such as a canal) of length L with gate structures placed at the inflow and outflow points as depicted in Figure 1. The canal cross section has a known geometry which is uniform along its length. Other geometrical features such as bed slope and maximum capacity/height are also known. Distance along the

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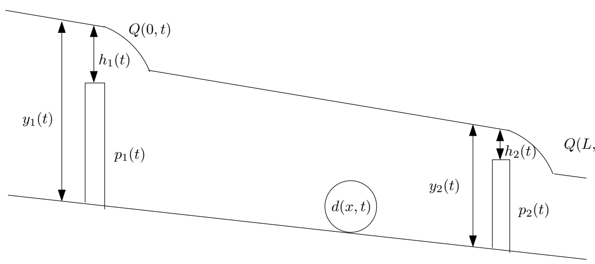

Fig. 1. Side view of a channel's pool along with upstream and downstream gates.

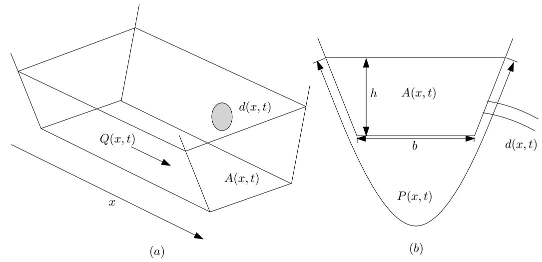

Fig. 2. (a) Open channel flow along a longitudinal axis indexed by abscissa x. (b) Cross-section of an open trapezoidal channel.

canal is parameterized as x. A point leak outflow or dump inflow is modeled by a function $d(x,t)$. The objective of this study is to detect the event d in both time and space. The ideal solution to such event detection is to be able to localize from sensors installed at gates. In automated downstream canal command systems (such as those found in Australia), water-level sensors and flow meters are installed for gate control. This problem has been studied in [2], [3]. However, the conclusion of this study is that while leak detection is an easy problem, leak localization is an inherently difficult problem with only gate sensor data.

In this paper, we take a fresh look at this problem from a fresh perspective. First, we hypothesize that by adding more sensors, leak localization can become remarkably easy problem to solve. Therefore, we study the performance of leak localization/detection in the presence of additional sensors installed at various lengths of the channel (and not necessarily at gates). Second, we note that addition of new sensors may result in significant budget and operational overhead. Therefore, there can not be an unconstrained addition of sensors to the system. Flow sensors, even when confined to the gates is an expensive and non-scalable option in a developing world setting. Therefore, we propose to detect leaks by instrumenting a channel with a minimum number of (inexpensive) water-level sensors only.

III. Numerical Simulation of Open Channel Flows

A. Saint Venant Equations

One dimensional, hyperbolic partial differential equations i.e. Saint-Venant equations[1] models open channel flows very well. Using the privilege of offtake factor $d(x,t)$ in the model, we can describe dynamics of water in canal in the presence of leaks or dumps. Eq. 1 describes the model:

$$\frac{\partial A}{\partial t} + \frac{\partial Q}{\partial x} = d,$$

$$\frac{\partial Q}{\partial t} + \left(\frac{gA}{B} - \frac{Q^2}{A^2}\right)\frac{\partial A}{\partial x} + \frac{2Q}{A}\frac{\partial Q}{\partial x} + gA(S_f - S_0) = \frac{Qd}{A}. \quad (1)$$

where, x is the distance variable coordinate, t is the time variable coordinate, A is the cross-sectional area of the channel, B is the width of water surface, Q is the flow (discharge), g is the gravitational constant and is considered $9.81m/s^2$, d is offtake, its value is negative for a leak and positive for dumps. S_f and S_0 are frictional and bed slope of the channel respectively. The frictional slope is modeled with the classical Manning formula, $S_f = \frac{Q^2 n^2}{A^2 R^{4/3}}$. Where, n is the Manning coefficient, R is the hydraulic radius, defined by $R = A/P$ and P is the wetted perimeter. Considering a trapezoidal channel, all above variables can be described well in Fig. 2.

[1]Model has been used extensively by hydraulic engineers and describes flows fairly accurately if supplied with geometry and operating conditions of the channel.

B. Numerical Solution of St. Venant PDEs

Due to the ease of implementation and availability of cheap computational resources, finite difference method has received most acceptance. Preissmann's Scheme is commonly accepted to be the most robust finite-difference scheme for solving the system of St. Venant's equations. The time, t, and spatial variable, x, are discretized in the Preissmann's scheme onto a grid on which the St. Venant's equations are approximated using approximations of partial derivatives presented in [11]. For ease of explanation, we restrict ourselves to rectangular channels in this paper. So channel's cross sectional area, $A(x,t)$, is $Bh(x,t)$, where B is the width of channel. Bottom slope, S_0, is approximated to be zero temporarily. Now using this setup, Eq. 1 can be represented in terms of flows and water levels as in Eq. 2, after replacing frictional slope S_f through Manning formula described before.

$$\frac{\partial h}{\partial t} + \frac{1}{B}\frac{\partial Q}{\partial x} = \frac{d}{B},$$

$$\frac{\partial Q}{\partial t} + gBh\frac{\partial h}{\partial x} + \frac{\partial(\frac{Q^2}{Bh})}{\partial x} + \frac{gn^2Q^2}{BhR^{\frac{4}{3}}} = \frac{Qd}{Bh}. \quad (2)$$

The system is solved for $0 \leq x \leq L$ and $t \geq 0$. Initial and final boundary conditions are given by

1) $h(x, t=0)$ and $Q(x, t=0)$ for $0 \leq x \leq L$.
2) $Q(x=0,t) = Q_0(t)$ and $Q(x=L,t) = Q_L(t)$ for $t \geq 0$.
3) $d(x=D,t)$ for $t \geq 0$.

Applying the Preissmann's scheme approximations to Eq. 2, we have

$$\begin{aligned}&\tfrac{1}{2}\left(\frac{h_i^{k+1}-h_i^k}{\Delta t} + \frac{h_{i+1}^{k+1}-h_{i+1}^k}{\Delta t}\right)+ \\ &\tfrac{1}{B}\left((1-\alpha)\left(\frac{Q_{i+1}^k-Q_i^k}{\Delta x}\right) + \alpha\left(\frac{Q_{i+1}^{k+1}-Q_i^{k+1}}{\Delta x}\right)\right) = \left(\tfrac{d}{B}\right)_P,\end{aligned} \quad (3)$$

$$\frac{1}{2}(\frac{Q_i^{k+1}-Q_i^k}{\Delta t}+\frac{Q_{i+1}^{k+1}-Q_{i+1}^k}{\Delta t})+gBh_P((1-\alpha)(\frac{h_{i+1}^k-h_i^k}{\Delta x})$$
$$+\alpha(\frac{h_{i+1}^{k+1}-h_i^{k+1}}{\Delta x}))+(1-\alpha)(\frac{(\frac{Q^2}{Bh})_{i+1}^k-(\frac{Q^2}{Bh})_i^k}{\Delta x})+$$
$$\alpha(\frac{(\frac{Q^2}{Bh})_{i+1}^{k+1}-(\frac{Q^2}{Bh})_i^{k+1}}{\Delta x})+(\frac{gn^2Q^2}{BhR^{\frac{4}{3}}})_P=(\frac{Qd}{Bh})_P. \quad (4)$$

In Eqs. 3 and 4, the water levels at node $k+1$ i.e. h_i^{k+1} and h_{i+1}^{k+1} and the flow rates at node $k+1$ i.e. Q_i^{k+1} and Q_{i+1}^{k+1} are unknown. This pair of equations can be written for all spatial grids i.e. $i = 1, 2, 3, ..., M-1$. We therefore obtain $2(M-1)$ algebraic equations with $2M$ unknowns. Two boundary conditions of flow rates complete the system as $Q_1^{k+1} = Q_0(t_{k+1})$ and $Q_M^{k+1} = Q_L(t_{k+1})$. This system of equations is solved using some iterative method like Newton-Raphson method. Detailed steps can be found from author's previous paper [12].

C. Simulation Results

Let us consider a (rectangular) channel pool. There is an upstream and downstream gate associated with this pool, In Fig. 1, variables $p_i(t)$, $y_i(t)$ and $h_i(t)$ (for $i = 1, 2, ..$) represent heights of the gate, water levels in the vicinity of gates and head over the gates respectively. For a sharp-edged rectangular gate, flow, Q, over any gate can be described by an empirical formula: $Q = 0.6\sqrt{g}bh^{3/2}$, where, g is acceleration due to Earth's gravity (in SI units) and b is the gate width. This relation can be used in describing inflows and outflows depending upon head-overs. We represent $0.6\sqrt{g}b$ by a constant c. So each pool has constants c_{in} and c_{out} corresponding to its upstream and downstream gates. Boundary conditions are derived from relation

$$Q_{in} = c_{in}h^{3/2}, \qquad Q_{out} = c_{out}h^{3/2}. \quad (5)$$

where, $h = y - p$ is used in calculating head over the gate. Over flows are set to zero if $h < 0$.

TABLE I

PHYSICAL DATA OF A RECTANGULAR POOL

Pool Parameters	Values
Length, $L(m)$	1000
Bottom Width, $b_0(m)$	5
Bottom Slope, S_0	0
Gravity, $g(m/s^2)$	9.81
Manning coefficient, n	00.02
$c_{in} = 0.6\sqrt{g}b_u$	4.8
$c_{out} = 0.6\sqrt{g}b_d$	3.6

Table I shows physical parameters of a rectangular channel. We have kept both gates closed so that there is neither any inflow nor outflow from pool. For simulation, we choose $\Delta x = 20$m and $\Delta t = 1$min. Initially at time $t = 0$min, water level is set at 0.7m and the channel is in a steady state. At time $t = 5$min, a leak starts at location $x = 340$m and continues till the end of simulation ($t = 30$min). Fall in water level is due to this leak leading to a rapid fall at $x = 960$m (Fig. 3). It can be seen clearly that the the water level is steady at $0.7m$ till about 7min after which level begins to drop. The 2 min delay can be explained by the distance traveled by water between measurement point (at

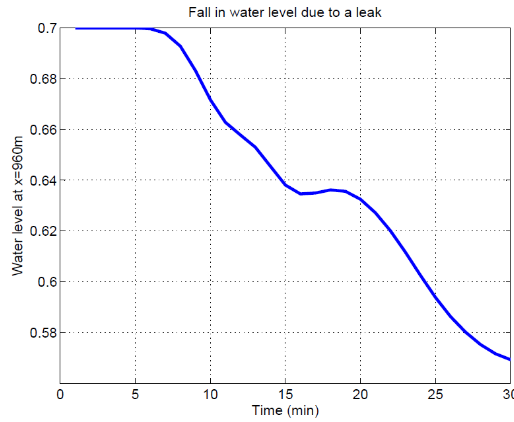

Fig. 3. Simulated response of water level due to a leak.

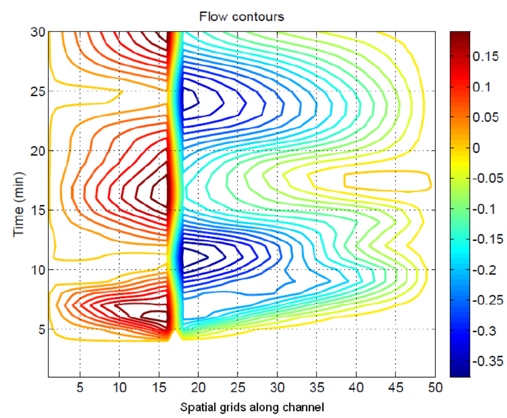

Fig. 4. Water flows in space-time window in the form of contours.

960m) and leak position (at 340m). At $t = 30$min the level drops to $0.57m$ and further continues till the pool empties.

Results of a full simulation is depicted in a 50×30 space-time window (i.e x, from 0 to $1000m$, with a simulation step of 20m, and t, from 0 to 30min, with a simulation step of 1min) in Fig. 4. The contour lines clearly mark the presence of a leak. Negative flow values after 17th spatial grid in Fig. 4) depict water traveling towards the leak. Similarly, we can simulate dumping scenario and useful results can be generated. All results in this paper hold true for dumps as well (viewed as negative leaks), unless explicitly mentioned.

IV. LEAK DETECTION IN TIME

We first decouple the problem of leak (dump) onset detection in time from localization in space. Consider two scenarios: Channel behavior with a leak event and another without any leaks, but keeping all initial and boundary conditions to be the same. From simulations, we see a clear difference in water levels for the two scenarios. For a pool with parameters given in Table I, the two scenarios are compared in Fig. 5. This time in addition to outflows in the form of a leaks at $t = 5$min, the pool is also excited by a gate inflow, Q_{in}, at $t = 10$min, from upstream gate in order to make simulations more interesting. Clearly, the heights

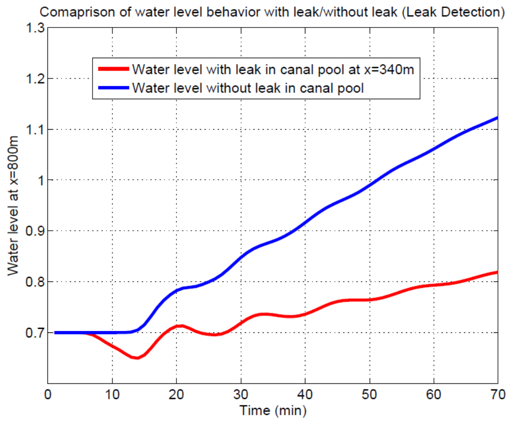

Fig. 5. Leak detection in time.

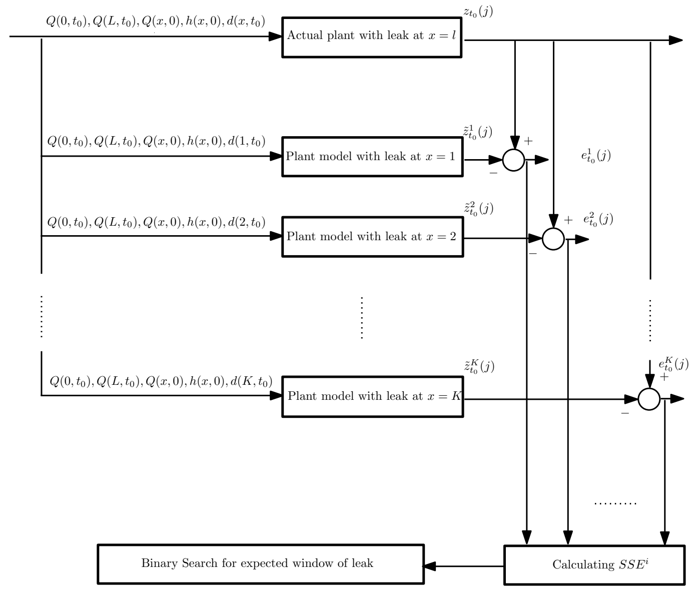

Fig. 6. Localization flowchart.

capture the onset of an outflow. In the leak scenario, the level drops at first in response to the leak outflow. However, when the effect of the gate inflow arrives at this location, the height begin to rise again. In the absence of the leak, there is only a rise. The difference between the two graphs clearly suggest the presence of a leak, detectable after roughly 15 minutes. The classical sequential analysis technique of cumulative sum (or CUSUM algorithm) can easily detect the start and ending of leak events from flow sensors installed at the gates. The algorithm is applied to a simplified volume balance model of the channel [3], [12]. Instead of solving the full PDE model, remarkable performance can be achieved by the volume balance model, at least for the detection problem, which is given by,

$$\frac{dy_2(t)}{dt} = c_{in}h_1^{3/2}(t-\tau) + c_{out}h_2^{3/2}(t) - s(t) - d(t),$$

where, τ is the end-to-end delay associated with channel flow, $s(t)$ represents scheduled off takes, $d(t)$, represents unauthorized off takes (i.e. leaks). We discretize the above mentioned equation and estimate $d(k) = d(kT)$ for discrete time step of $T = 1$min to make a profile.

$$d(k) = y_2(k+1) - y_1(k) - c_{in}h_1^{3/2}(k - \lceil\tau\rceil) - c_{out}h_2^{3/2}(k) - o(k). \tag{6}$$

Eq. 6 estimates leak for each discrete step. If there is no leak, the terms should balance to zero. However, if a leak is present $d(k)$ becomes more and more negative. By fixing a threshold γ on a cumulative sum (CUSUM) i.e. $\sum_{k=\lceil\tau\rceil}^{n} d(k) \lesseqgtr \gamma$, as discussed in [3]. one can detect the leaks at time $t = nT$. The threshold can be tuned to incorporate losses from factors such as evaporation or seepage.

V. Localization with Temporal Knowledge

By using the detection algorithm from previous section, we proceed with an accurate estimate of leak onset and finish times $d(t)$. We assume that there is only one singular leak at an unknown location x_0. Let the channel be modeled by Eq. 1. The state of the system at time t is given by

$$\mathbb{X}(x,t) = \begin{pmatrix} y(x,t) \\ Q(x,t) \end{pmatrix}, \quad 0 \le x \le L.$$

.

1) Assume perfect knowledge of $d(t)$, boundary conditions for all times and that the system initializes from a steady-state at $t = 0$. This gives us a good knowledge of the initial state of the system $\mathbb{X}(x,0), 0 \le x \le L$, from boundary conditions [9] alone.
2) Install K water-level sensors along the canal at (possibly irregular) spacing. A sensor j, located at position x_j records the height at a particular time slot t_0 to give us reading as, $z_{t_0}(j) = h(\mathbb{X}(x_j,t_0)) + \upsilon_{t_0}(x_j), \quad 1 \le j \le K$. Where, h represents the sensor model and $\upsilon_{t_0}(x_j)$, is the measurement noise at time instant t_0. We ignore this noise for the time being.
3) Run K number of PDE models using the same initial state $\mathbb{X}(x,0)$ and boundary conditions while assuming leaks at sensor locations x_i, $i = 1,\ldots,K$. Predict the expected readings for model i at all sensor locations as, $\tilde{z}^i_{t_0}(j) = h(\mathbb{X}^i(x_j,t_0)), \quad 1 \le j \le K$. Where $\mathbb{X}^i(x_j,t_0)$ is the simulated state of the channel at t_0 sec at sensor location x_j, assuming a leak present at location x_i.
4) Calculate the error between predicted measurements and actual measurements for each model as, $e^i_{t_0}(j) = z_{t_0}(j) - \tilde{z}^i_{t_0}(j)$.
5) Calculate a Sum of Squared Errors, SSE^i, for each model as, $SSE^i = \sum_{j=1}^{K}(e^i_{t_0}(j))^2$.
6) Run the Leak-Search algorithm (see description below) on SSE^i data to predict the true location of the leak.

These steps are captured in a flow chart given in Fig. 6.

A. Simulation Results

Again consider a canal pool with parameters described in Table I, but having length of the pool doubled now i.e. $L = 2000m$. Introduce a leak at $x = 1300m$. The leak is comparable to the outflow of a typical domestic tube well, simulating a possible water theft scenario. 20 water

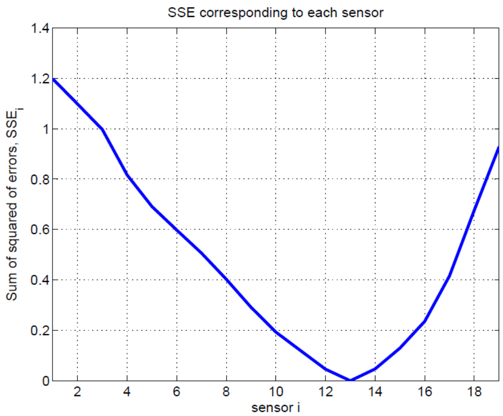

Fig. 7. Leak localization. Leak is placed exactly at sensor location.

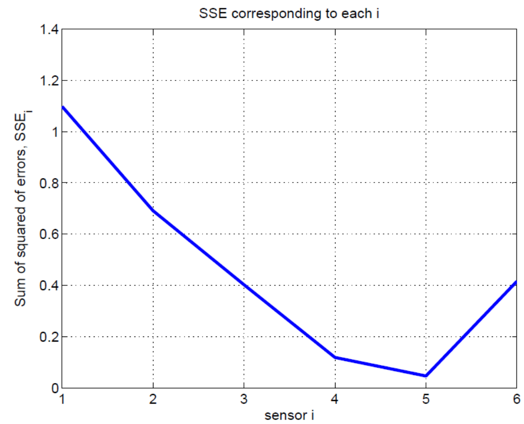

Fig. 8. Leak localization. Leak is not co-located with any sensor.

level sensors are placed at a regular spacing of $100m$. This places the leak exactly at the location of the 13^{th} sensor. By following the steps outlined in the flowchart of Fig. 6, we get an SSE^i graph with a minimum at model 13. Note that this matches the leak position exactly. Furthermore, the error minimum can be interpreted as a maximum likelihood estimate of the location by running all possible scenarios with equal probability. Note, that in this case the graph is symmetrical about the min point up to a large deviation. Therefore, the errors associated with other models hardly give any more information on top of the maximum likelihood estimate. Therefore, there is no need to do further processing when the leaks are located exactly at the sensor locations. However, when the leaks are not co-located with sensors (which is more likely), one can think of using data from other models to estimate a trend by which resolution can be improved. We give below the rationale and working of one such method, the Leak-Search algorithm.

B. Leak-Search Algorithm

Consider the same scenario as above with a leak at 1300m, except that we reduce the number of level meters to 6, placed regularly at a mutual distance of $300m$. Simulations are repeated for 6 models. The plot of Fig. 8 shows the results of leak localization for this scenario. Note that a maximum likelihood estimate, estimates the leak to be at the location of sensor 5, i.e. position 1500m with an error of 200m. However, also note that the error is not exactly equal to zero. Further, the error at model 4 is very close to the global minimum and much less than other models. This hint towards resolving the error somewhere between sensor 4 and 5. The LeakSearch algorithm is a binary search process that helps us in reducing window size of expected region of leak along the channel using errors from other models.

The LeakSearch algorithm takes all sensors and their sum of squared of errors as input and provides an improved estimate of the leak location as well as the most probable window of the leak region. By running this algorithm we can reduce expected window size, $\Delta(x)$, from that of the crude max likelihood estimate i.e. $\frac{L}{K-2}$ to $\frac{L}{K(K-2)}$. This relation has been derived from the analysis of the LeakSearch algorithm and verified from simulation tests. We can also manipulate this equation to find the number of sensors required for a desired leak detection resolution by solving K in $K^2 - 2K - \frac{L}{\Delta x} = 0$. Note that there is a quadratic improvement in accuracy as compared to a linear maximum likelihood estimate based on sensor spacing alone.

VI. Localization with Model Uncertainty and Noisy Sensors

In this section, we investigate the robustness of our method in the presence of errors in modeling as well as sensor noise. There are many sources of disturbances therefore it becomes very important to be able to implement the system without the need to *tune* the estimator for each real scenario.

A. Model Uncertainty

There can be errors in the approximation of channel geometry and other physical parameters. For example, channel width can not be constant through out the channel or not exactly modeled. Similarly the manning coefficient can also change value place to place. The bed slope of the channel can also show variation. Therefore, such uncertainties can create deviation from the predictions of an ideal scenario. Taking the example from Table I, but having length of the pool doubled again i.e. $L = 2000m$, we have varied channel width up to 30 percent from its true value with a leak at 1300m and 20 equispaced sensors. The error curves show a remarkable resilience against this model uncertainty. The minimum can be discerned easily at the correct value. Similar simulations have been run for other parameter variations and it has been found that the method is quite robust.

B. Noisy Sensors

Now, let us consider noise in sensor measurements. It can be expected to have an imprecision of a few inches in a cheap water level sensor. We artificially inject a normal i.i.d noise with zero mean and standard deviation of σ inches to the sensors, i.e. $\upsilon_t \sim \mathcal{N}(0, \sigma^2)$. Once again, using the same channel as mentioned in previous scenarios, we perform some simulations by placing the sensors at every $100m$ along the channel. In Fig. 10, the errors are calculated for

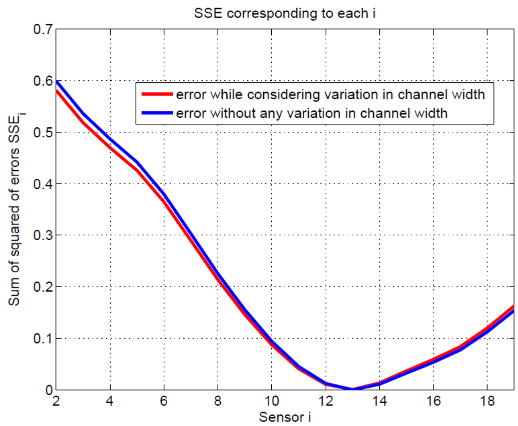

Fig. 9. Localization under performance to model uncertainty.

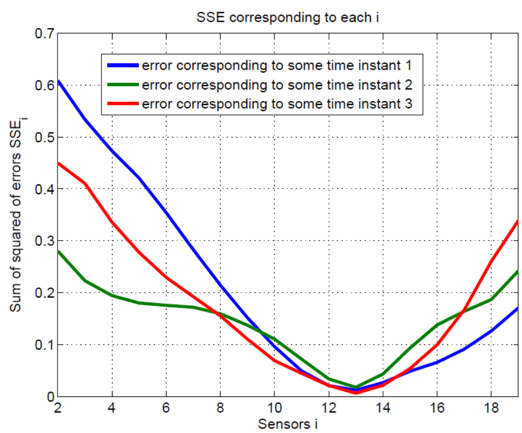

Fig. 10. Error curves at different time steps.

sensor readings taken at different time instants but with a fixed sensor noise of standard deviation equal to 1 inch. In all cases, the minimum is marked very clearly at the true location of $x = 13 \times 100 = 1300m$. Fig. 9 shows various error curves with various standard deviations of noise. Again, the simulations show a remarkable resilience against noise. Even under a very severe degradation of sensor noise close to half a foot, the estimated leak is not off by more than a 100m, thus showing a graceful degradation. Of course, the constraint on sensing accuracy can be met by other factors such as taking multiple measurements and averaging.

VII. CONCLUSIONS

A framework for leak or dump detection and localization in open channel flows has been developed. The method works in real scenarios where there is model uncertainty, noise and costs associated with instrumentation and automation. The requirements on sensing grow sub-linearly with accuracy, thus ensuring reasonable scalability with respect to infrastructure requirements. Future and ongoing work by the authors involve the improvement in localization with prior information such as leakage/dumping history, a *Bayesian Inversion* framework for detailed incorporation of all model and sensing uncertainties and a formal analysis of robustness and sensor scaling.

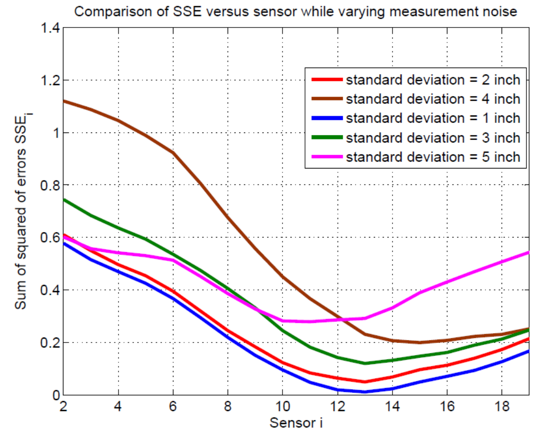

Fig. 11. Localization performance degradation with sensor noise.

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