When do robots become hyperbolic?

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Outline

• Complex networks as reconfigurable systems
• The idea of a configuration space
• A simple cooperative localization problem
• Sensor arrangements on robots
• Configuration spaces
• Managing topological complexities
• Conclusions
Reconfigurable systems / Dynamic Networks

- Lab on a chip
- Coordinated drop movement

Courtesy: Duke digital microfluidics

Courtesy: RPI robotics
Reconfigurable systems / Dynamic Networks

• Robotic warehousing

• KIVA Systems video (0-45 sec)
  http://www.youtube.com/watch?v=lWsMdN7HMuA#
Reconfigurable systems / Dynamic Networks

- Biology: Ant colonies, bird flocks, fish schools, etc.
Common themes …

• Complex systems are made up of a large number of agents
• Agents interact via sensing and communication
• Agents are either controlled centrally or they make collective decisions (distributed systems)
Common themes ....

- Agents are constrained to occupy “positions”
- Agents cannot bump into each other (collision avoidance)
- The positions of all agents collectively make a configuration
Configuration Space

• Set of all configurations is a \textit{configuration space}
• \textit{Reconfiguration}: Moving from one configuration to another
Example

- Two agents
- Agents live in a plane (free space)
- One agent is fixed
- Distance between the two agents always remain the same
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- Configuration space is a circle.
- One configuration is an angle $\theta$
- Reconfiguration is moving on a circle
Example 2

- A bit more complicated
- Distance between the two agents can vary between two distances
Example 2

- A bit more complicated
- Distance between the two agents can vary between two distances
Example 2

- A bit more complicated
- Distance between the two agents can vary between two distances

- Configuration space is a fat circle
- Each Configuration is given by \((d, \theta)\)
Example 3

- 3 agents, one fixed
- Two distances fixed

- Configuration space again is a fat circle
- Configuration is given by two angles \((\theta_1, \theta_2)\)
Example 3

• 3 agents, one fixed
• Two distances fixed

• Configuration space, again, is a fat circle
• Configuration is given by two angles \((\theta_1, \theta_2)\)
Example 4

- What is the configuration space?
Challenge

- How to go from one configuration to the other

\[(\theta_1, \theta_2) \quad \rightarrow \quad (\phi_1, \phi_2)\]

Looking for guarantees and algorithms about reconfigurable systems
Cooperative Localization

Achieve $r_1$, $r_2$, $r_3$ while
1. Maintaining full network connectivity
2. Avoid collisions

$r_1 < R_c$, $r_2 < R_c$, $r_3 < R_c$,
$r_1 > R_o$, $r_2 > R_o$, $r_3 > R_o$,
$r_1 + r_2 - r_3 < 0$,
$r_1 - r_2 - r_3 < 0$,
$r_1 - r_2 + r_3 < 0$. 
What sensors to fit on robots?

Radio

Camera / bearing

GPS

Range finder

GPS

Range finder

Radio
What sensors to fit on robots?

- radio
- camera / bearing
- GPS
- range finder
- radio
What arrangement is better?

2 broadcasts needed before localization

1 broadcast needed before localization

No communication needed
A configuration space

- **Allowed triplets** \((r_1, r_2, \theta)\) under constraints of collision avoidance and network connectivity

![Diagram showing homeomorphism and solid double torus](image)
Some rubber sheet geometry (topology)
Some rubber sheet geometry (topology)
How to reconfigure / re-localize?

- Motion planning on a topologically complex space
- Covering space preserves geodesics but is topologically simple

- **Step 1:** cut open the holes to get the fundamental domain
- **Step 2:** glue repeatedly to get covering space
How to reconfigure / re-localize?

Tessellation of the Poincare disc by the fundamental domain via Mobius transformations

\[ G_q := \begin{pmatrix} i \cosh(x_0) \exp(i\alpha) & -i \sinh(x_0) \\ i \sinh(x_0) & -i \cosh(x_0) \exp(-i\alpha) \end{pmatrix} \]

a slice of the thickened Poincare disc
Poincare Disc

\[ X \]

\[ \mathbb{R}^2 \]
Reconfigurable systems & Negative Curvature

• Configuration spaces of all “reconfigurable systems” have (locally) negative curvature

• Deep connection between geometry, topology and reconfiguration
Conclusions

- Reconfigurable systems
- Complex evolving networks
- Fundamental research
- Beautiful mathematics