When do robots become hyperbolic?

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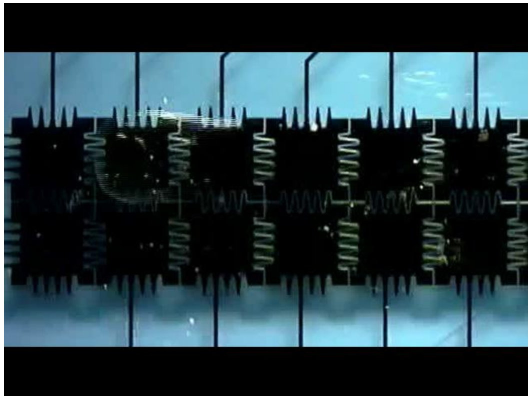
Outline

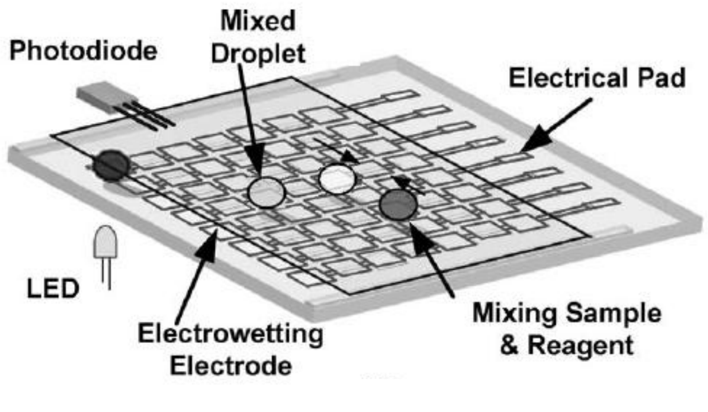
- Complex networks as reconfigurable systems
- The idea of a configuration space
- A simple cooperative localization problem
- Sensor arrangements on robots
- Configuration spaces
- Managing topological complexities
- Conclusions

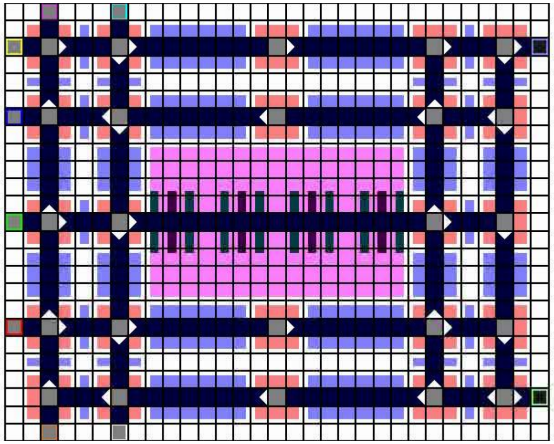
Reconfigurable systems / Dynamic Networks

- Lab on a chip
- Coordinated drop movement



Courtesy: Duke digital microfluidics

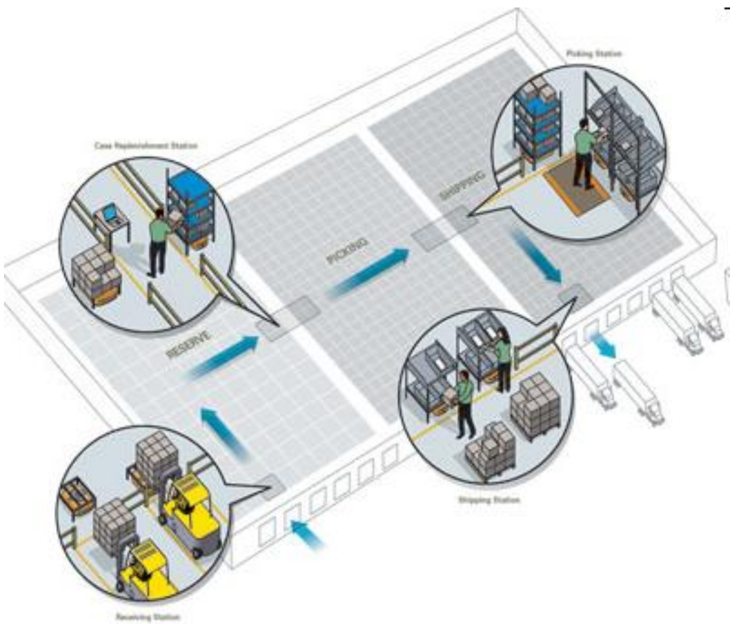




Courtesy: RPI robotics

Reconfigurable systems / Dynamic Networks

- Robotic warehousing



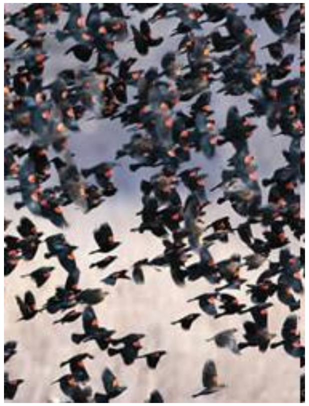
Robots zoom around a warehouse. The robots, which can carry either 1,000 pounds or 3,000 pounds, travel on tracks and are manufactured by Kiva Systems of Woburn, Mass.

- KIVA Systems video (0-45 sec)

http://www.youtube.com/watch?v=lWsMdN7HMuA#

Reconfigurable systems / Dynamic Networks

- Biology : Ant colonies, bird flocks, fish schools ….

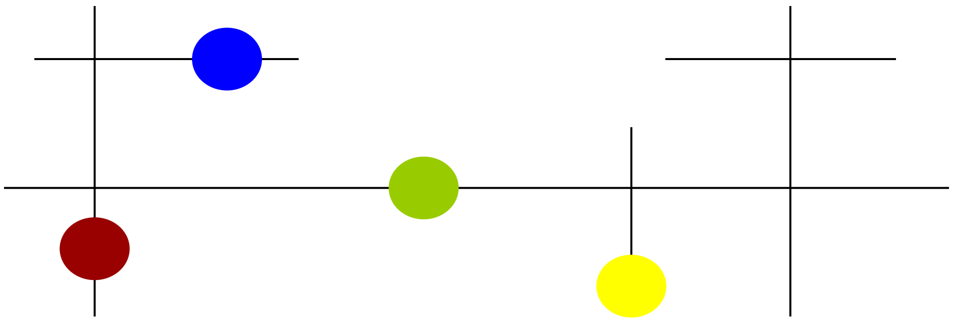


Common themes …

- Complex systems are made up of a large number of agents
- Agents interact via sensing and communication
- Agents are either controlled centrally or they make collective decisions (distributed systems)

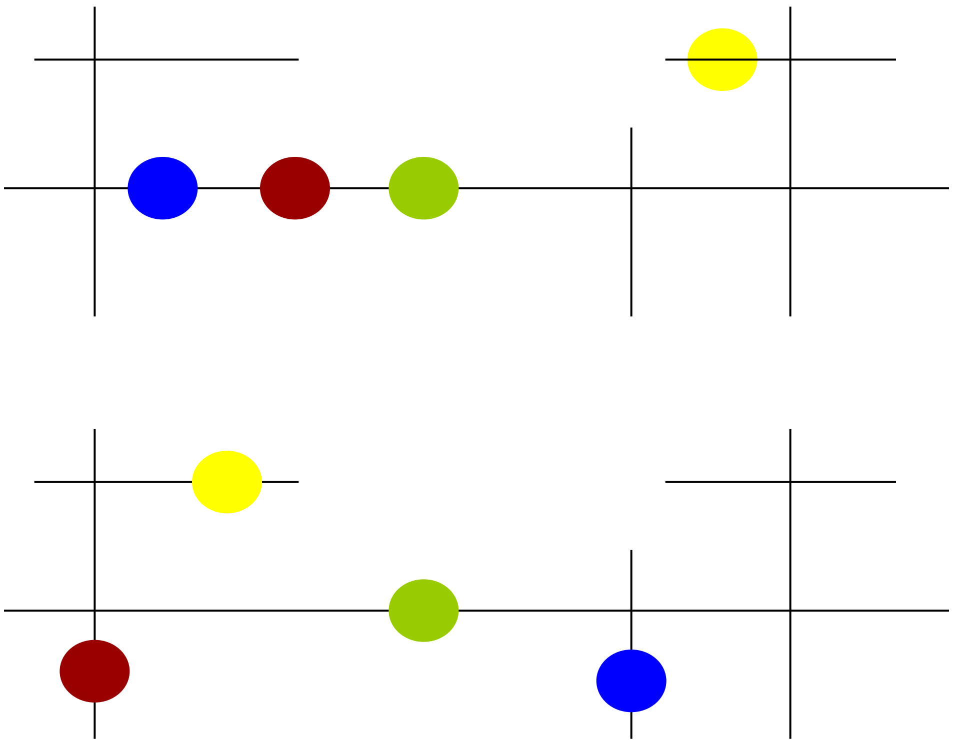
Common themes ….

- Agents are constrained to occupy "positions"
- Agents cannot bump into each other (collision avoidance)
- The positions of all agents collectively make a *configuration*



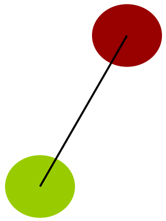
Configuration Space

- Set of all configurations is a *configuration space*
- *Reconfiguration:* Moving from one configuration to another



Example

- Two agents
- Agents live in a plane   (free space)
- One agent is fixed
- Distance between the two agents always remain the same



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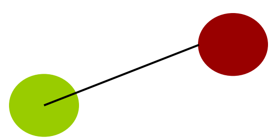

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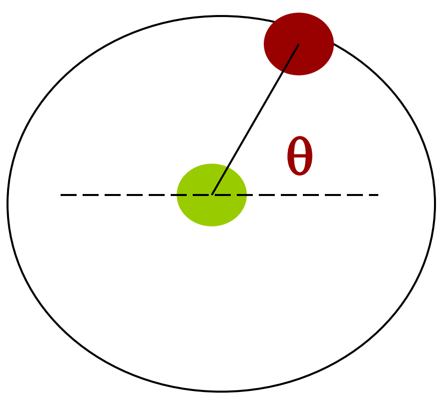
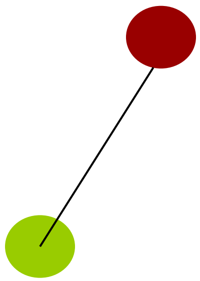


- Configuration space is a circle.
- One configuration is an angle θ
- Reconfiguration is moving on a circle

Example 2

- A bit more complicated
- Distance between the two agents can vary between two distances



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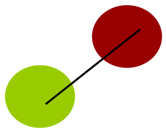

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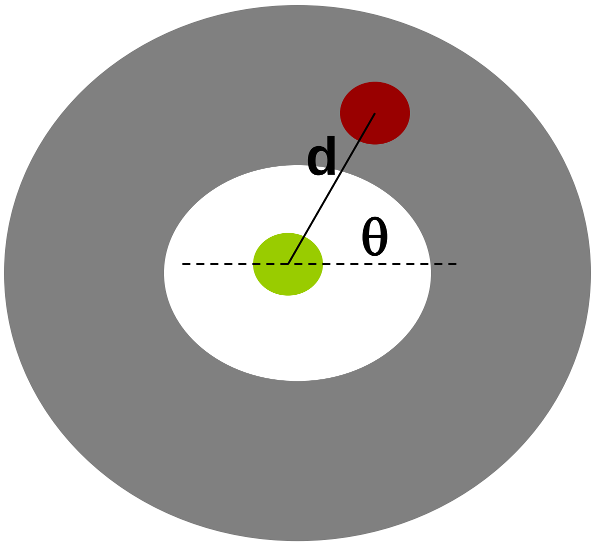
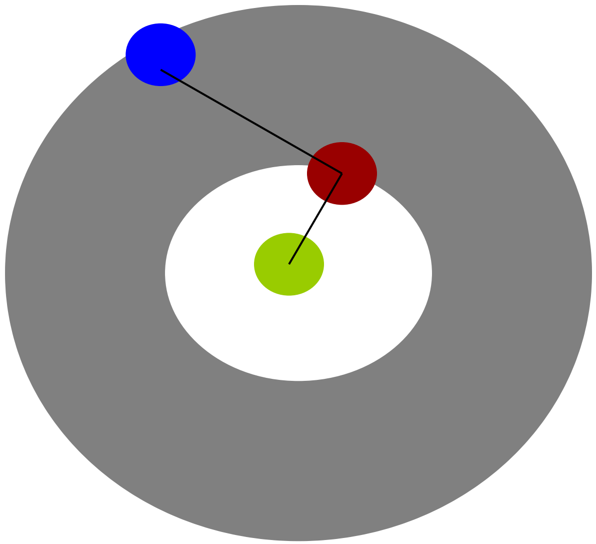


- Configuration space is a fat circle
- Each Configuration is given by (**d**, θ)

Example 3

- 3 agents, one fixed
- Two distances fixed



- Configuration space again is a fat circle
- Configuration is given by two angles (θ_1, θ_2)

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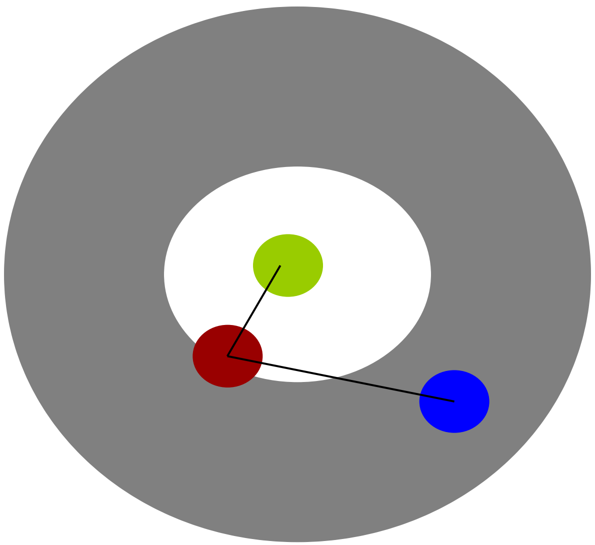
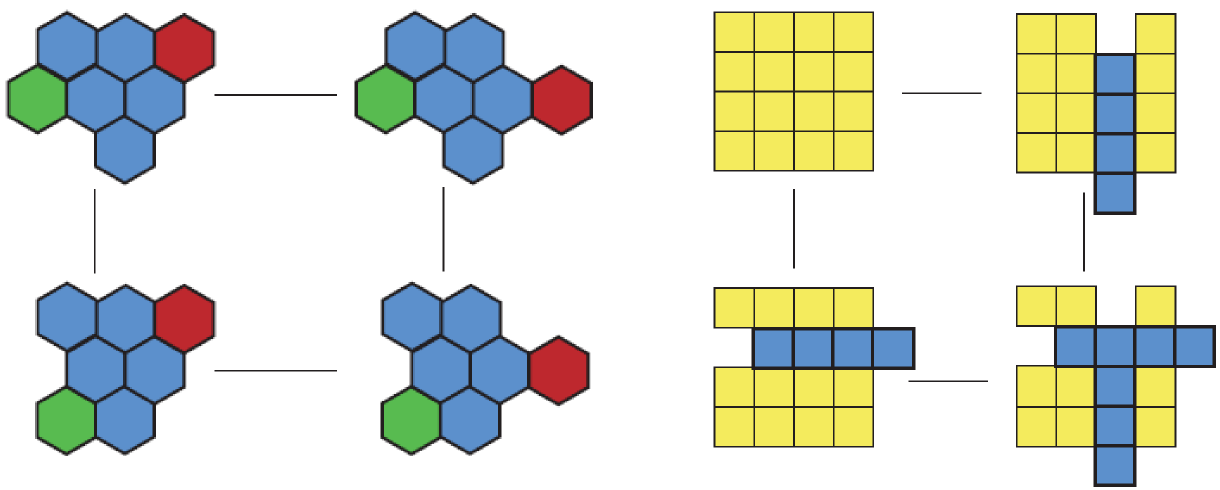


- Configuration space, again, is a fat circle
- Configuration is given by two angles (θ_1, θ_2)

Example 4

- **What is the configuration space?**



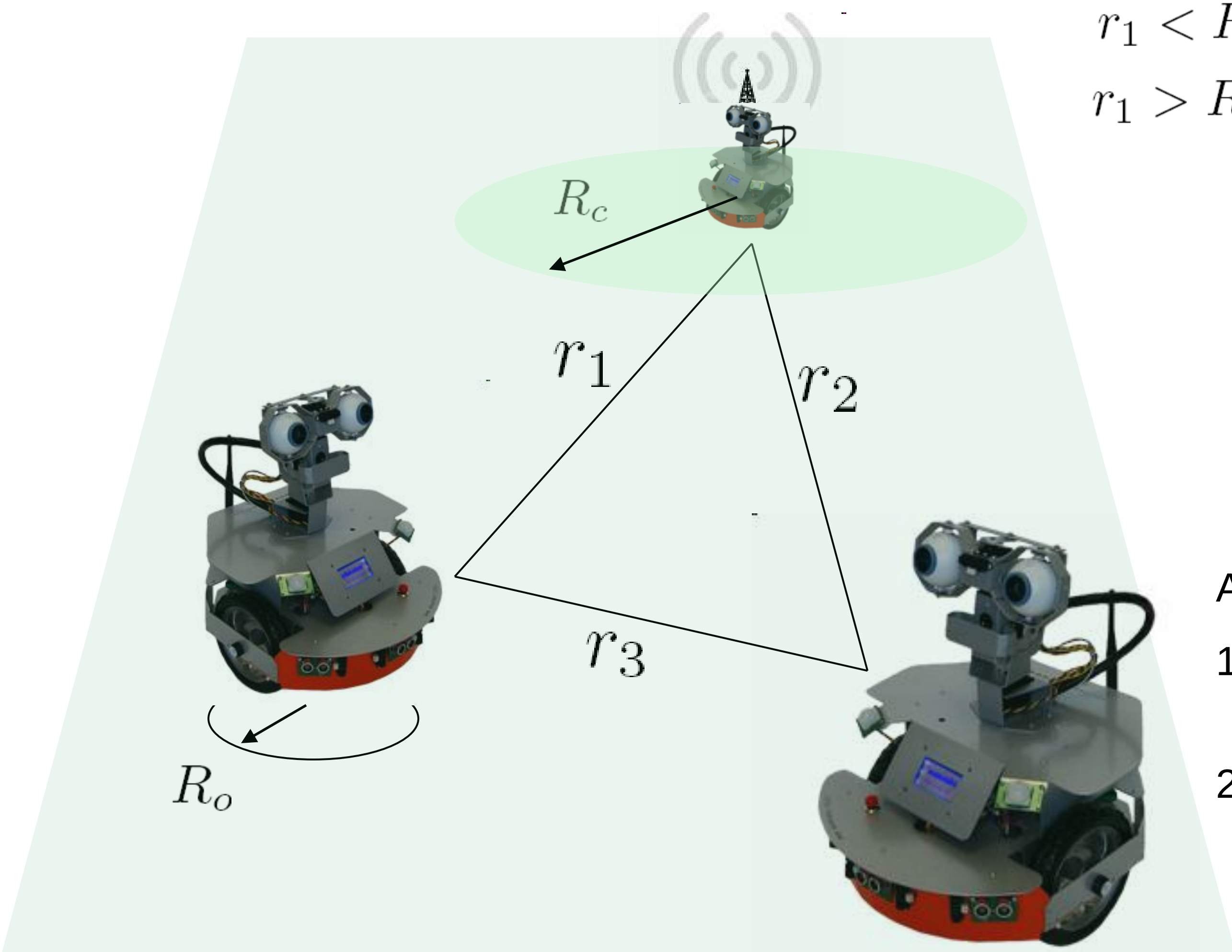
Challenge

- **How to go from one configuration to the other**

$$(\theta_1, \theta_2) \longrightarrow (\varphi_1, \varphi_2)$$

Looking for gaurantees and algorithms about reconfigurable systems

Cooperative Localization

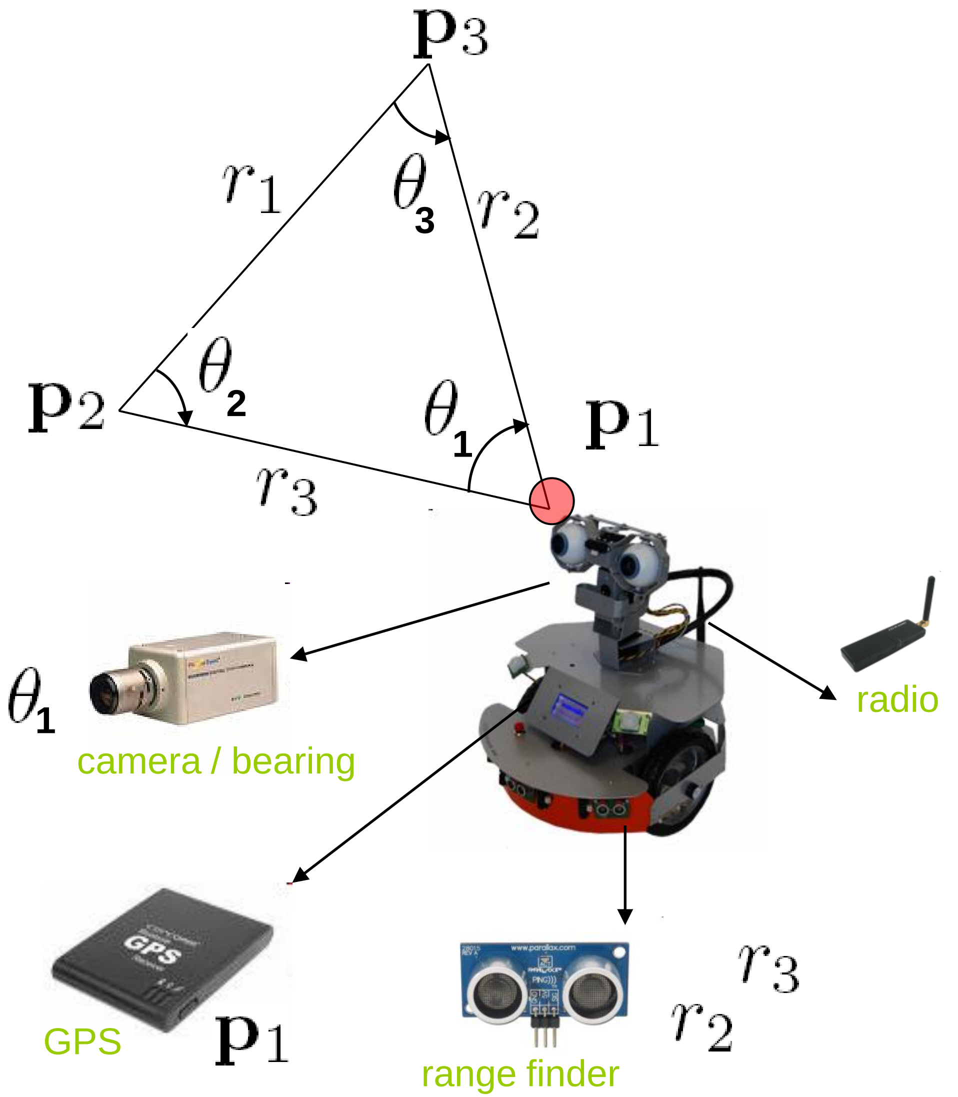


$$r_1 < R_c,\ r_2 < R_c,\ r_3 < R_c,$$
$$r_1 > R_o,\ r_2 > R_o,\ r_3 > R_o,$$
$$r_1 + r_2 - r_3 < 0,$$
$$r_1 - r_2 - r_3 < 0,$$
$$r_1 - r_2 + r_3 < 0.$$

Achieve r_1, r_2, r_3 while

1. Maintaining full network connectivity
2. Avoid collisions

What sensors to fit on robots?



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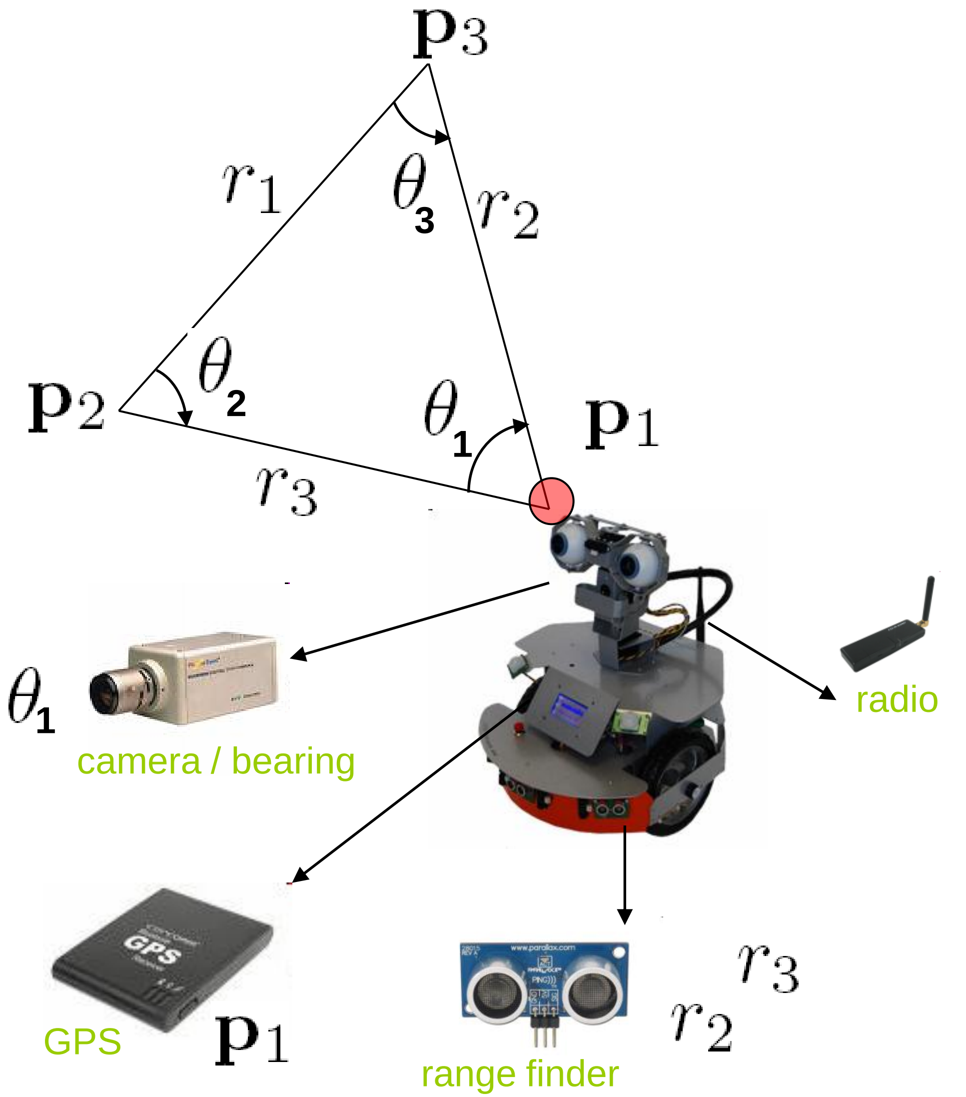
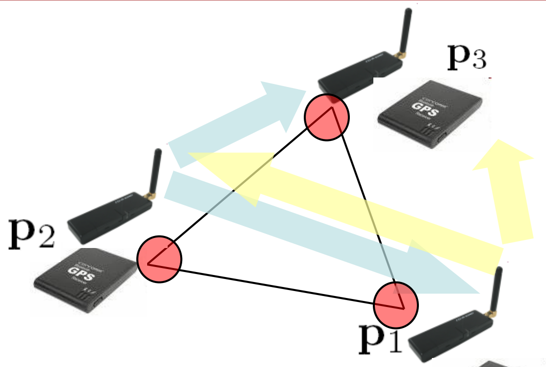
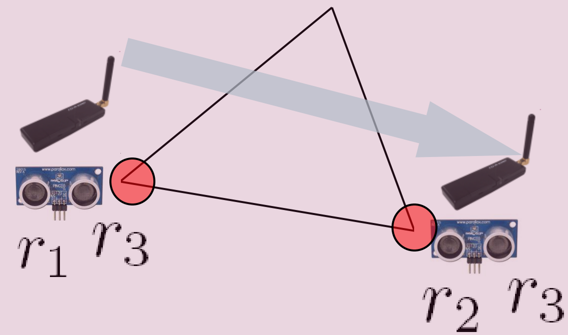


What arrangement is better?

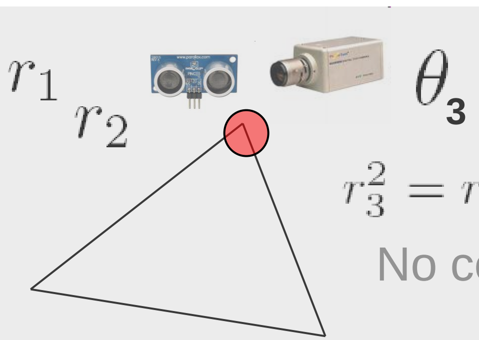


$$r_1 = \|\mathbf{p}_2 - \mathbf{p}_3\|$$

2 broadcasts needed
before localization



1 broadcast needed
before localization

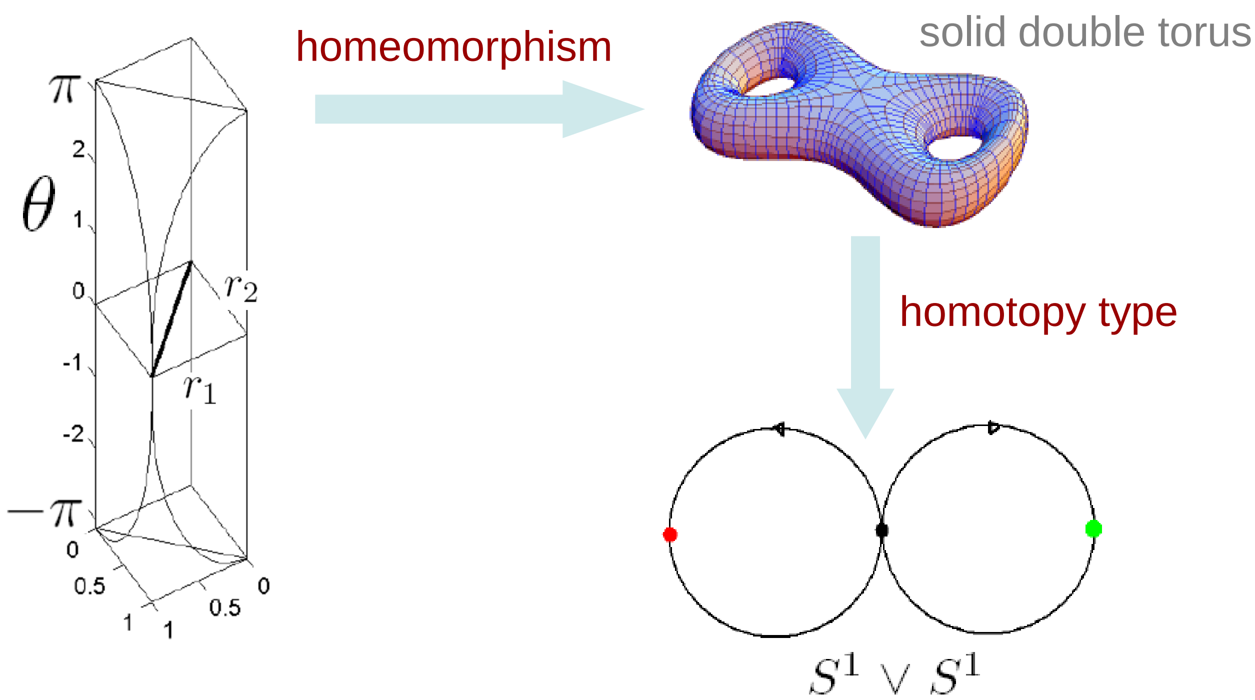


$$r_3^2 = r_1^2 + r_2^2 - 2 r_1 r_2 \cos(\theta)$$

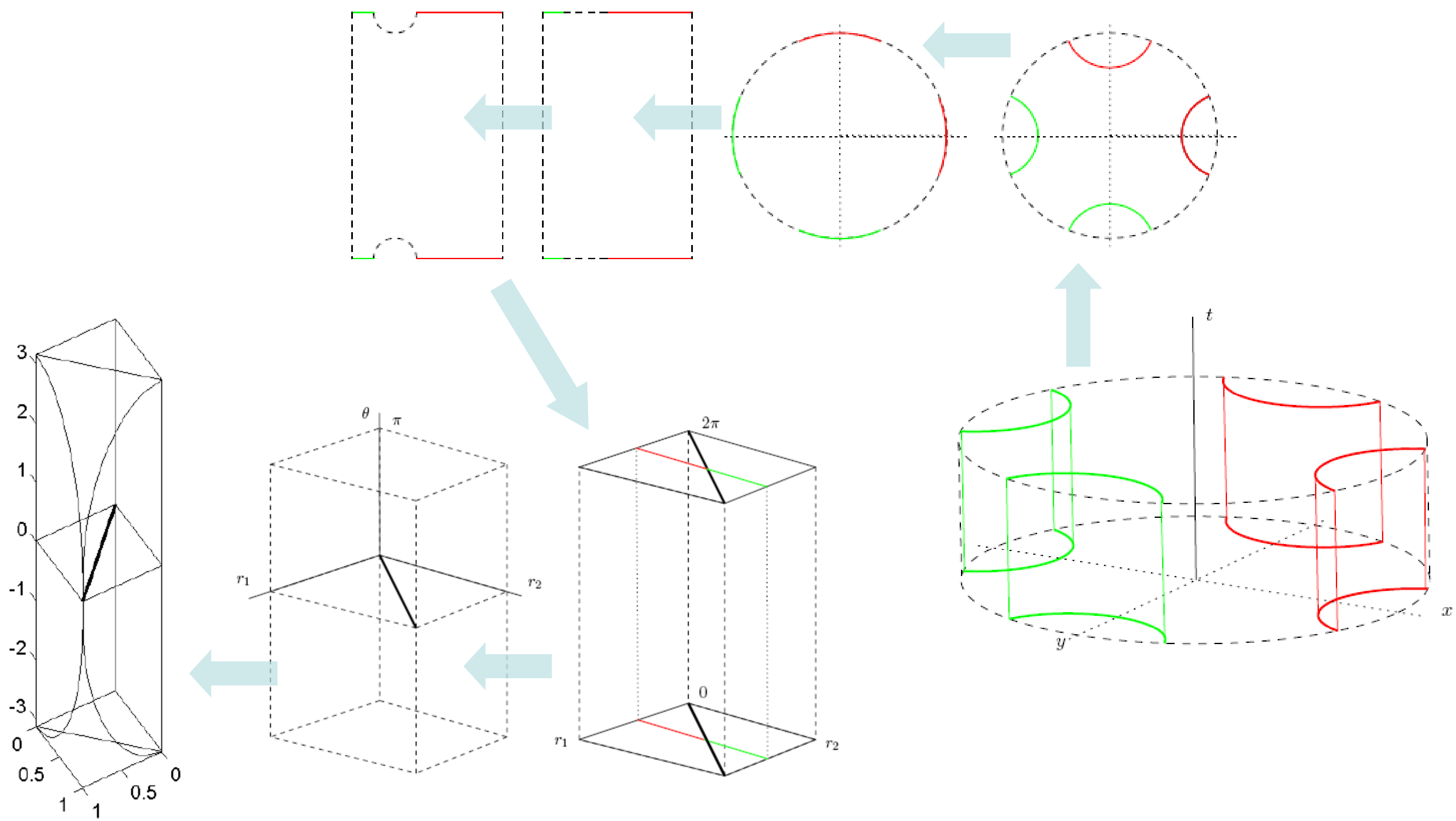
No communication needed

A configuration space

- **Allowed triplets (r_1, r_2, θ) under constraints of collision avoidance and network connectivity**



Some rubber sheet geometry (topology)



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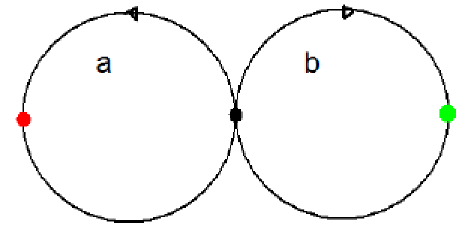
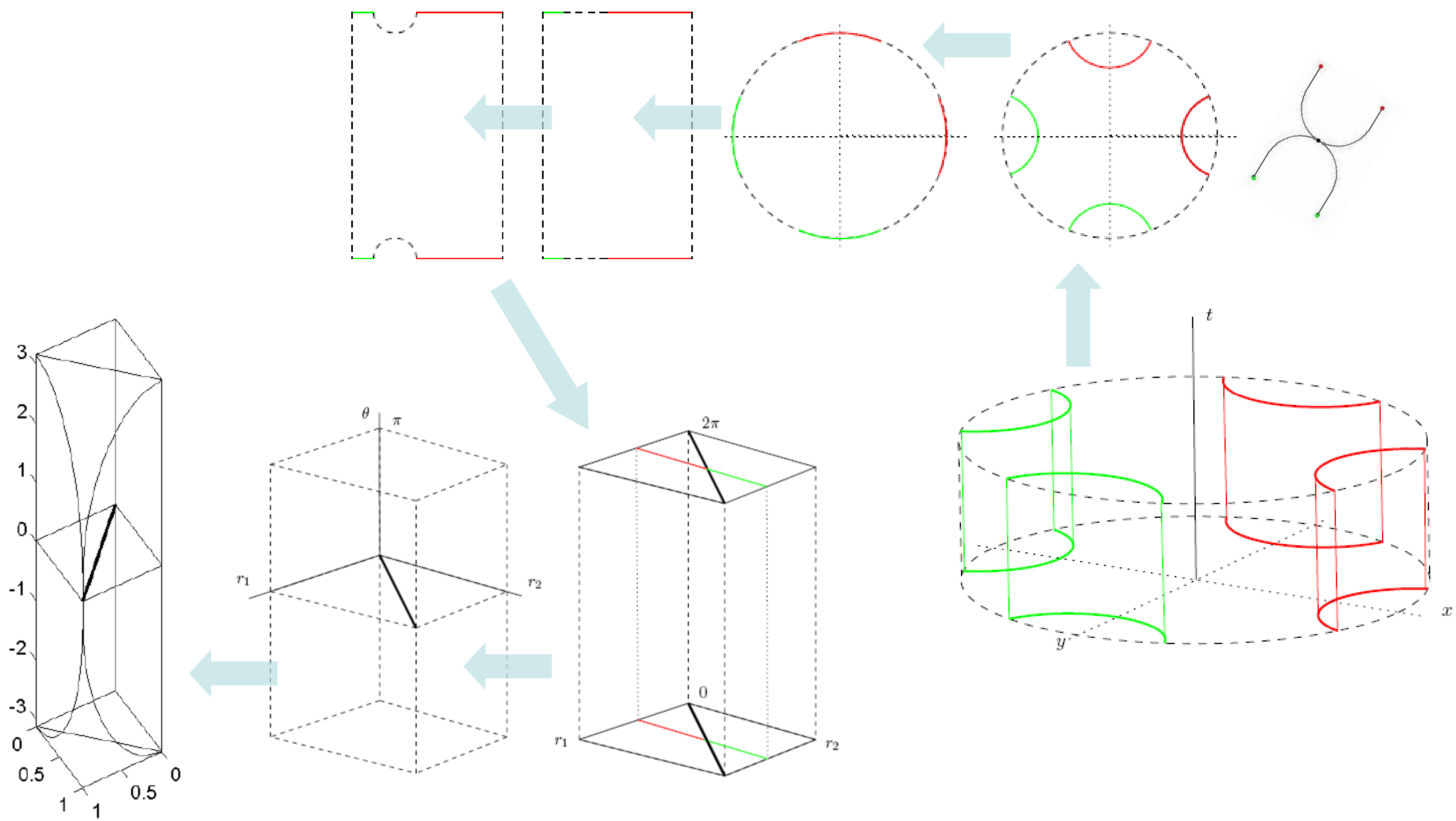
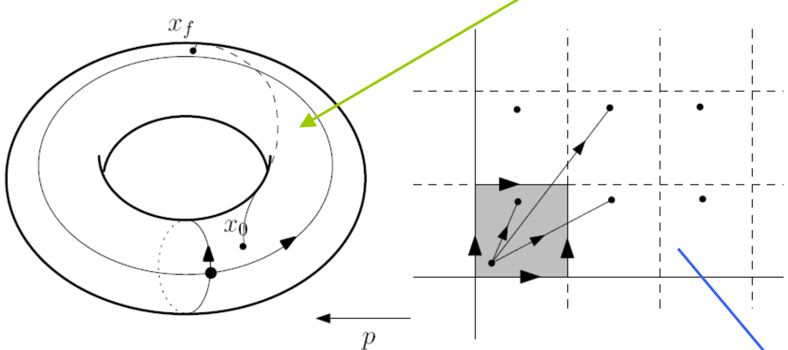


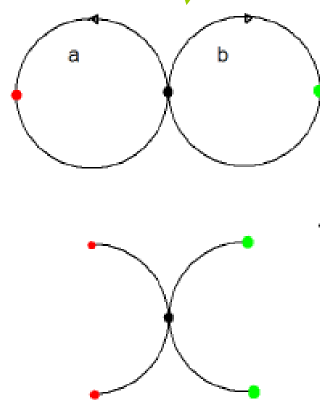


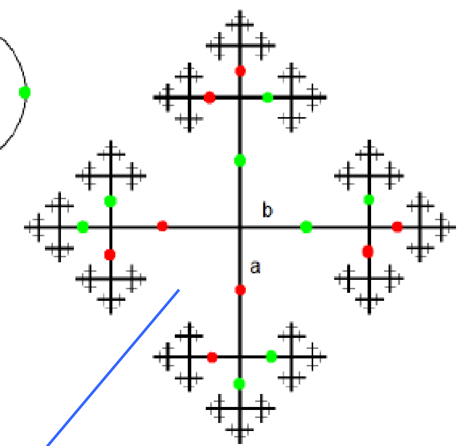
How to reconfigure / re-localize?

- Motion planning on a topologically complex space
- Covering space preserves geodesics but is topologically simple

Step 1: cut open the holes to get the fundamental domain



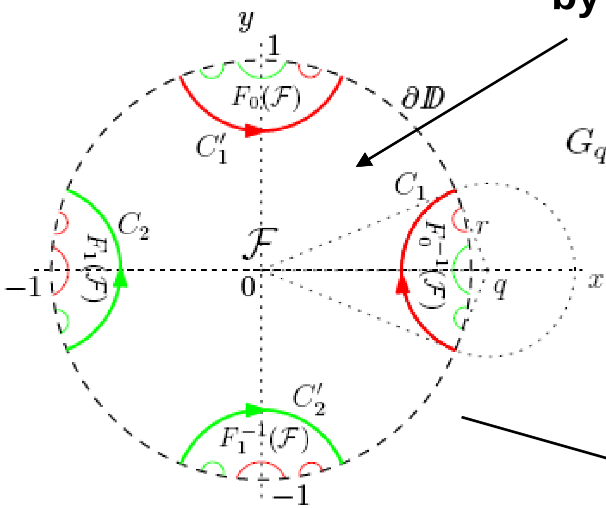




Step 2: glue repeatedly to get covering space

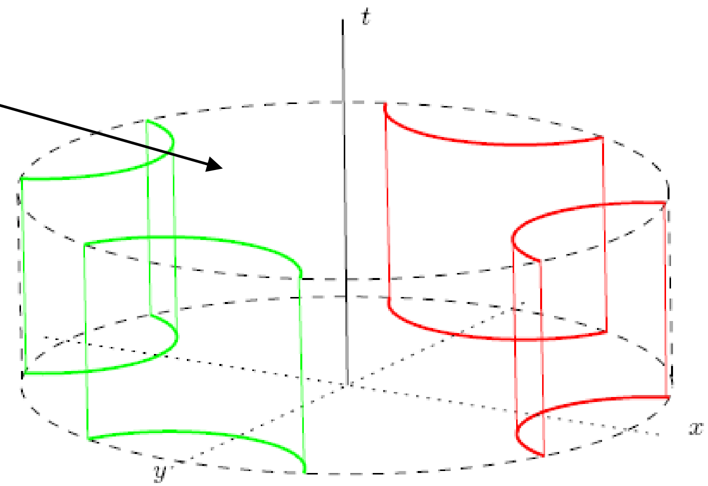
How to reconfigure / re-localize?

Tessellation of the Poincare disc by the fundamental domain via Mobius transformations

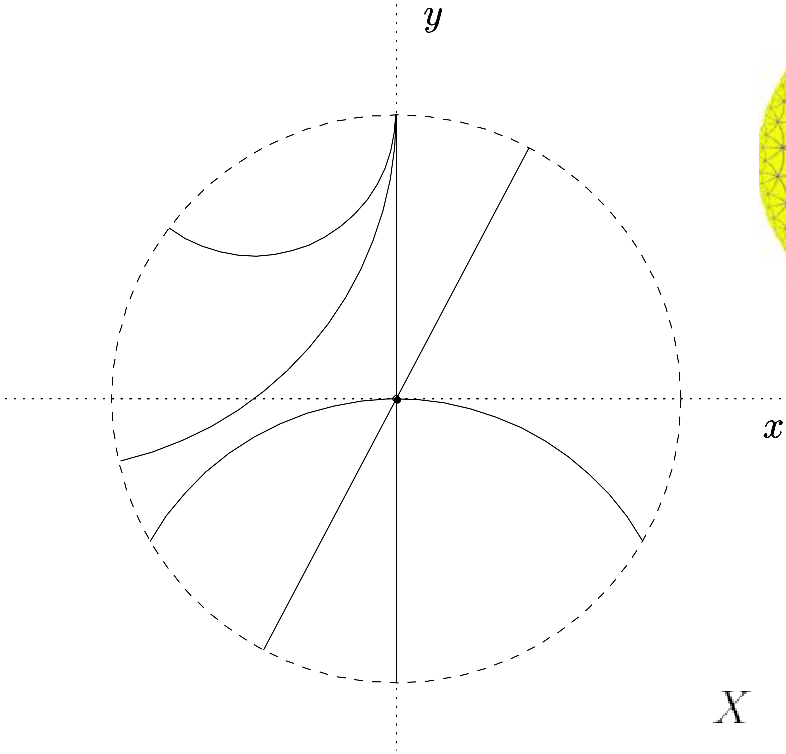


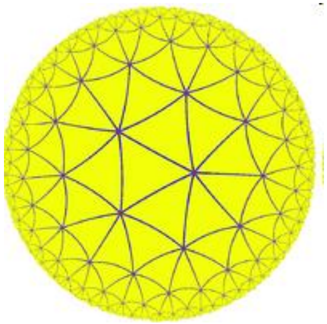
$$G_q := \begin{pmatrix} i\cosh(x_0)\exp(i\alpha) & -i\sinh(x_0) \\ i\sinh(x_0) & -i\cosh(x_0)\exp(-i\alpha) \end{pmatrix}$$

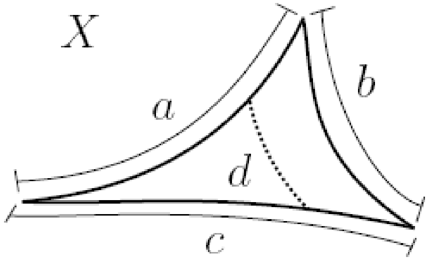
a slice of the thickened Poincare disc

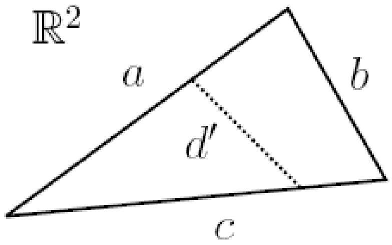


Poincare Disc









Reconfigurable systems & Negative Curvature

- Configuration spaces of <u>all</u> "reconfigurable systems" have (locally) negative curvature

- Deep connection between geometry, topology and reconfiguration

Conclusions

- Reconfigurable systems
- Complex evolving networks
- Fundamental research
- Beautiful mathematics