CMPE-432. Feedback Control Systems.
Homework #2.*

Spring 2010.

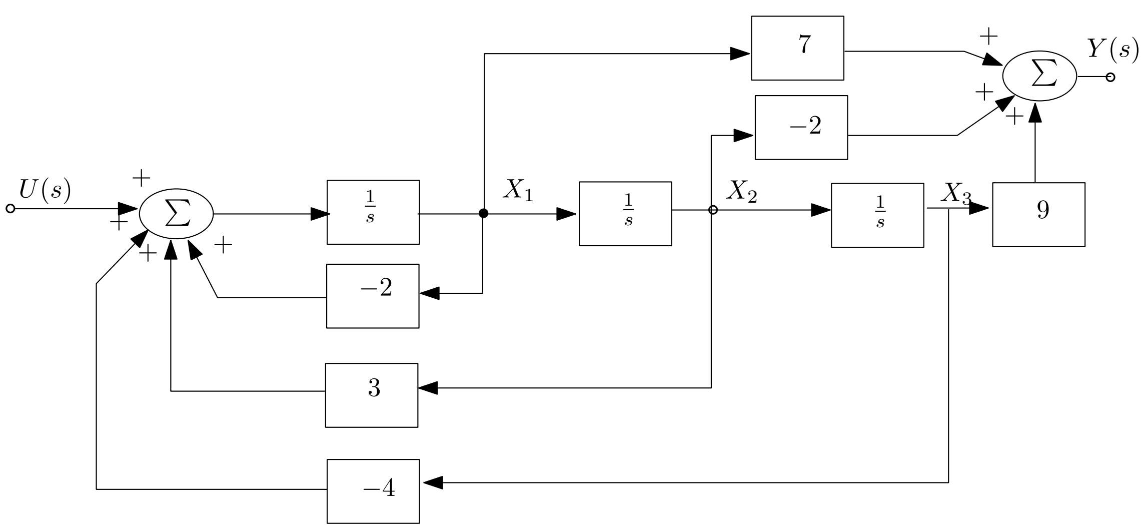
Due date March 3^{rd}, 2010.

System Response and Stability based problems:

Problem 1

Compute the transfer function for the block diagram shown in figure. Also:

1. Write the third order differential equation that relates y and u.
2. Write three simultaneous first order differential equations using variable x_1, x_2 and x_3 as defined in the block diagram.



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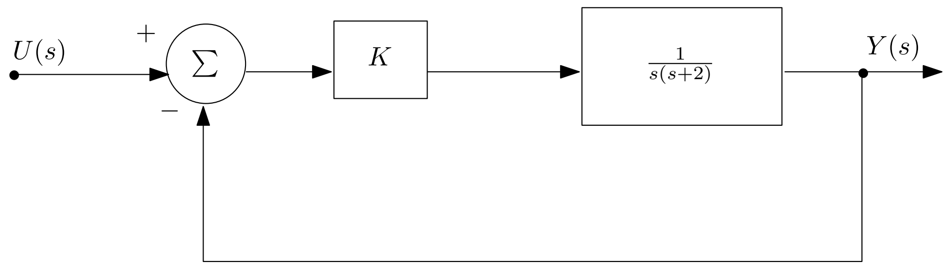
Problem 2

For the unity feedback system shown in figure. Specify gain K of the proportional controller so that the output $y(t)$ has an overshoot of no more than 5 percent in response to a unit step.



Problem 3

The equation of motion for the DC motor is given as:

$$J_m\ddot{\theta}_m + (b + \frac{K_t K_e}{R_a})\dot{\theta}_m = \frac{K_t}{R_a}v_a.$$

Assuming that:

$$J_m = 0.01 kg.m^2.$$

$$b = 0.001 N.m.sec.$$

$$K_e = 0.02 V.sec.$$

$$R_a = 10\Omega.$$

$$K_t = 0.02 N.m/A.$$

1. Find the transfer function between the applied v_a and the motor speed $\dot{\theta}_m$.
2. What is the steady-state speed of the motor after a voltage $v_a = 10V$ has been applied?
3. Find the transfer function between the applied voltage v_a and the shaft angle θ_m.
4. Suppose feedback is added to the system in part 3. so that it becomes a position servo device such that the applied voltage is given by:

   $$v_a = K(\theta_r - \theta_m).$$

   where K is the feedback gain. Find the transfer function between θ_r and θ_m.
5. What is the maximum value of K that can be used if an overshoot $M_p < 20$ percent is desired?
6. What values of K will provide a rise time of less than 4 sec?
7. Plot the step response of the position servo system for $K = 1$ using MATLAB. Find the overshoot and rise time from plot.

Problem 4

Consider the following second-order system with an extra pole:

$$H(s) = \frac{w_n^2 p}{(s+p)(s^2+2\zeta w_n s+w_n^2)}.$$

Show that the unit step response is:

$$y(t) = 1 + Ae^{-pt} + Be^{-\sigma t}\sin(w_d t - \theta).$$

where:

$$A = \frac{-w_n^2}{w_n^2-2\zeta w_n p+p^2},$$

$$B = \frac{p}{\sqrt{(w_n^2-2\zeta w_n p+p^2)(1-\zeta^2)}},$$

$$\theta = \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{-\zeta}\right) + \tan^{-1}\left(\frac{\sqrt{1-\zeta^2}}{p-\zeta w_n}\right).$$

1. Which term dominates $y(t)$ as p gets large?
2. Give approximate values for A and B for small values of p.
3. Which term dominates as p gets small?

Problem 5

Use Rouths stability criterion to determine how many roots with positive real parts the following equations have:

1. $s^5 + 10s^4 + 30s^3 + 80s^2 + 344s + 480 = 0.$
2. $s^4 + 6s^2 + 25 = 0.$

Problem 6

Find the range of K for which all the roots of the following polynomial are in the LHP.

$$s^5 + 5s^4 + 10s^3 + 10s^2 + 5s + K = 0.$$