FORMATION CONTROL UNDER LIMITED SENSORY RANGE CONSTRAINTS

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Abstract

Based on the assumption that all robots in a given multi-agent scenario can evaluate a global formation function, we show how a model independent coordination strategy for multi-agent formation control can be obtained. The main theorem states that under a bounded tracking error assumption our method stabilizes the formation error. We furthermore complement this result with an investigation of how limited sensory range capabilities affect the group performance.

1 Introduction

In the maturing field of mobile robot control, a natural extension to the traditional trajectory tracking problem [5, 8, 10, 19] is that of *coordinated tracking*. In its most general formulation, the problem is to find a coordinated control scheme for multiple robots that make them maintain some given, possibly time-varying, formation at the same time as the robots, viewed as a group, executes a given task. The possible tasks could range from exploration of unknown environments, where an increase in numbers could potentially reduce the exploration time, navigation in hostile environments where multiple robots make the system redundant and thus robust [2], to coordinated path following [6]. The latter of these tasks is applicable in manufacturing or construction situations where multiple robots are asked to carry or push objects in a coordinated fashion [13, 16].

The formation problem for multiple robots has been extensively studied in the literature, and, for instance, in [2] a behavior based, decentralized control architecture is exploited, where each individual platform makes sure that it is placed appropriately with respect to its neighbors. In [6, 7], the situation is slightly different and the solution is based on letting one robot take on the role of the leader, meaning that all of the other robots position themselves relative to that robot. Furthermore, in [12, 22] an extensive line of work has been conducted with the dynamic model taken into account explicitly, while a very specific type of "string stability" is achieved for multiple autonomous vehicles.

In contrast to this, the approach suggested in this paper is both platform independent, provenly successful, and general enough to support a number of different actual control designs. The idea that we capitalize on is that the tracking controllers can be designed independently of the coordination scheme, which provides us with some additional freedom in the control design. Our ambition is thus that the strategy, proposed in this paper, should be thought of as an abstract coordination principle rather than a solution to a very specific multi-agent problem, as illustrated in [9]. Furthermore, the proposed globally stable coordination strategy is defined with respect to formations of the form

$$F(\mathbf{x}_1, \ldots, \mathbf{x}_m) = \sum_{i=1}^{m} \sum_{j \neq i} \tau_{ij} (\|\mathbf{x}_i - \mathbf{x}_j\|^2 - a_{ij}^2)^2, \tag{1}$$

where $\mathbf{x}_i \in \mathbb{R}^n$, $i = 1, \ldots, m$, are the individual robot states, $\tau_{ij} = \tau_{ji}$ are scalar weights that determine how important it is that robot i and j are located appropriately relative to each other, and $a_{ij} = a_{ji}$ are the desired relative distances. In Section 2 it will be shown that by generating desired trajectories for the individual robots, using a steepest descent method, then, as long as the real robots track their reference trajectories well enough (the tracking error should be bounded), F can be driven asymptotically to a (non-unique with respect to group rotations) global minimum.

However, it is clearly not always the case that the various members of the robot team are within sensory range of each other, and in this paper we investigate what effect *limited range sensing capabilities* have on the stability of the group. This bounded range sensor capability will be the topic of Section 3, and the sensory capabilities will be modeled as simple thresholding functions, where robot i has a knowledge of the state of robot j only if $\|\mathbf{x}_i - \mathbf{x}_j\| \leq \delta_i$. This relation will not be assumed to be symmetric, i.e. δ_i is not necessarily equal to δ_j, since it is conceivable that different robots may have different sensing capabilities. It will be shown that this limited sensory capability has direct implications on the global performance of the multi-agent group. The main results obtained in Section 3 is thus a study of the different conditions under which the following three different formation behaviors emerge:

1. Global formation keeping can be achieved.
2. The formation consists of one weakly connected component.
3. The formation consists of disconnected components.

By *weakly connected* we understand that all robots in the team can be observed implicitly, i.e. robot 1 can observe robot 2, that can observe robot 3, and so on, and these results directly imply that perfect formation keeping under bounded range sensing capabilities is not always possible. In fact, in Section 3.3 we briefly outline how the local interactions (e.g. *social potential fields* [14, 20, 21]) should be defined in the presence of bounded sensory capabilities.

2 Global Formation Control

In this section we follow the development in [9], and the multi-agent system is given by m mobile robots, each of which is governed by its own set of system equations

$$\dot{\mathbf{x}}_i = f_i(\mathbf{x}_i, \mathbf{u}_i), \tag{2}$$

where $\mathbf{x}_i \in \mathbb{R}^n$ is the state of the ith robot and $\mathbf{u}_i \in \mathbb{R}^k$ is the control. The m robots should keep a certain relative position and orientation, as specified by the formation function in (1), while moving along one given path, specified for the *virtual leader*, e.g. the geometric center of the formation.

We thus need to allow for the possibility of having a moving formation since we want the virtual leader to follow a given reference path. If the desired path that we want the virtual leader to follow is given by $\mathbf{p}_0(\cdot)$, we choose to parameterize the trajectory for the virtual leader, $\mathbf{x}_0 \in \mathbb{R}^n$, as

$$\mathbf{x}_0(t) = \mathbf{p}_0(s_0(t)), \tag{3}$$

where $s_0 \in \mathbb{R}$ is a function of t (time), and where we assume that the trajectory is smooth, i.e. $\|\frac{\partial \mathbf{p}_0(s_0)}{\partial s_0}\| \neq 0$ for all s_0.

The reason for calling $\mathbf{x}_0$, together with its dynamics, a virtual leader (see for example [9]) is because it takes on the role of the leader for the formation. Using this terminology, our additional task is to design m new virtual robots for the individual robots to follow. We are thus free to design the evolution of these additional virtual vehicles, and we ignore the question concerning how to actually track these new virtual vehicles within the context of this paper. (See for example [5, 8, 10, 19] for a representative sample of the literature on asymptotically stable trajectory tracking.)

In light of the previous paragraph, it is more convenient to consider a moving frame with coordinates centered at $\mathbf{x}_0$. In the new coordinates we thus have $\tilde{\mathbf{x}} = \mathbf{x} - \mathbf{x}_0$. Let the desired trajectories (subscript d), or virtual vehicles, be defined in the moving frame by

$$\begin{array}{l} \tilde{\mathbf{x}}_{id} = \tilde{\mathbf{p}}_i(s_i),\ i = 1, \ldots, m \\ \dot{\tilde{\mathbf{x}}}_{id} = \frac{\partial \tilde{\mathbf{p}}_i(s_i)}{\partial s_i} \dot{s}_i, \end{array} \tag{4}$$

where we have not yet specified what the desired trajectories should look like. In fact, $\frac{\partial \tilde{\mathbf{p}}_i(s_i)}{\partial s_i}$ and $\dot{s}_i \in \mathbb{R}$ can be designed by us, and they should be chosen in a systematic fashion so that the formation constraint is respected.

The solution we propose is to let the desired trajectories be given by the steepest descent direction to the desired formation, i.e., we set

$$\frac{\partial \tilde{\mathbf{p}}(\mathbf{s})}{\partial \mathbf{s}} = -\nabla F(\tilde{\mathbf{x}}_d), \tag{5}$$

where we have grouped together the contributions from the different robots as

$$\begin{array}{l} \nabla F(\tilde{\mathbf{x}}_d)^T = (\frac{\partial F(\tilde{\mathbf{x}}_d)}{\partial \tilde{\mathbf{x}}_{1d}}^T, \ldots, \frac{\partial F(\tilde{\mathbf{x}}_d)}{\partial \tilde{\mathbf{x}}_{md}}^T) \\ \tilde{\mathbf{p}}(\mathbf{s})^T = (\tilde{\mathbf{p}}_1^T(s_1), \ldots, \tilde{\mathbf{p}}_m^T(s_m)) \\ \mathbf{s}^T = (s_1, \ldots, s_m) \\ \tilde{\mathbf{x}}_d^T = (\mathbf{x}_{1d}^T - \mathbf{x}_0^T, \ldots, \mathbf{x}_{md}^T - \mathbf{x}_0^T). \end{array} \tag{6}$$

The idea now is to let the evolution of the different virtual vehicles be governed by differential equations containing error feedback in order to make the control scheme robust. This can be viewed as a combination of the conventional trajectory tracking, where the reference trajectory is parameterized in time, and a dynamic path following approach [19], where the criterion is to stay close to the geometric path, but not necessarily close to an *a priori* specified point at a given time.

In order to accomplish this, we define the evolution of the reference points as

$$\dot{s}_i = ce^{-\alpha_i \rho_i},\ i = 1, \ldots, m, \tag{7}$$

where $c,\ \alpha_i > 0$ and $\rho_i = \|\mathbf{x}_i - \mathbf{x}_{id}\| = \|\tilde{\mathbf{x}}_i - \tilde{\mathbf{x}}_{id}\|$. We furthermore want the motion of s_0 to capture how well the formation is being respected. For this we define

$$\rho_a = \sum_{i=1}^{m} \rho_i \tag{8}$$

and set

$$\dot{s}_0 = \frac{c_0}{\|\frac{\partial \mathbf{p}_0(s_0)}{\partial s_0}\|} e^{-\alpha_0 \rho_a}, \tag{9}$$

where $c_0,\ \alpha_0 > 0$.

With these designs we have the following stability theorem:

Theorem 2.1 (Coordinated Tracking and Formation Control) *Under the assumption that the real robots track their respective reference trajectories perfectly, it holds that*

$$\lim_{t \to \infty} F(\tilde{\mathbf{x}}_d) = 0. \tag{10}$$

Proof:

$$\frac{d}{dt} F(\tilde{\mathbf{x}}_d) = \nabla F(\tilde{\mathbf{x}}_d)^T \dot{\tilde{\mathbf{x}}}_d = -\sum_{i=1}^{m} \|\frac{\partial F(\tilde{\mathbf{x}}_d)}{\partial \tilde{\mathbf{x}}_{id}}\|^2 ce^{-\alpha_i \rho_i}. \tag{11}$$

Now assume that we have perfect tracking, i.e. $\rho_i = 0,\ i = 1, \ldots, m$. This assumption, combined with the that, by definition, F is positive definite and convex, implies that $\frac{d}{dt}F(\tilde{\mathbf{x}}_d)$ is negative definite. This concludes the proof. ∎

Corollary 2.1 *If all the tracking errors are bounded, i.e. it holds that $\rho_i \leq \rho < \infty$, $i = 1, \ldots, m$, then*

$$\lim_{t\to\infty} F(\tilde{\mathbf{x}}_d) = 0. \tag{12}$$

The proof of this corollary is just a straight forward extension of the proof of the previous theorem. This corollary is furthermore very useful since one typically does not want $\rho = 0$ due to the potential chattering that such a control strategy might give rise to [8]. Instead it is desirable to let $\rho > 0$ be the look-ahead distance at which the robots should track their respective reference trajectories.

Example 2.1 (Triangular Formations) *In order to illustrate the usefulness of the proposed coordination strategy, we consider a triangular formation without the orientation fixed:*

$$\begin{aligned} F(\tilde{\mathbf{x}}) &= \\ &= (\|\tilde{\mathbf{x}}_1 - \tilde{\mathbf{x}}_2\|^2 - 1)^2 + (\|\tilde{\mathbf{x}}_2 - \tilde{\mathbf{x}}_3\|^2 - 1)^2 \\ &+ (\|\tilde{\mathbf{x}}_3 - \tilde{\mathbf{x}}_1\|^2 - 1)^2 + (\|\tilde{\mathbf{x}}_1\|^2 - \frac{1}{3})^2 \\ &+ (\|\tilde{\mathbf{x}}_2\|^2 - \frac{1}{3})^2 + (\|\tilde{\mathbf{x}}_3\|^2 - \frac{1}{3})^2, \end{aligned}$$

which corresponds to maintaining an equilateral triangular shape (side lengths equal to one) between the different robots. (One of the terms in the function is actually redundant for defining the shape.) The mid-point of the triangle is the virtual leader in this case. An example of this can be seen in Figure 1.

As pointed out in [13, 16, 17] such rigid body formations are useful in a number of applications where groups of robots are asked to carry or push objects in a coordinated manner.

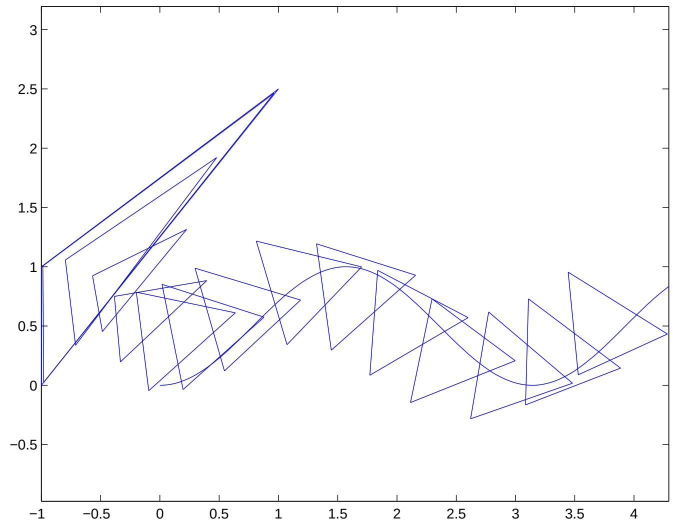


Figure 1: The evolution of a triangular formation under a perfect tracking assumption. The triangular formation and the reference path for the mid-point of the triangle are shown.

3 Bounded Range Sensory Capabilities

The assumption that (4) and (7) can be evaluated exactly by all individual robots is clearly unreasonable. Not only does that assumption rely on that each robot has perfect knowledge of where all the different robots are located, but also that this knowledge can be obtained, i.e. that the range sensors can measure long enough distances. In this section we assume that robot i has a knowledge of the state of robot j only if $\|\mathbf{x}_i - \mathbf{x}_j\| \leq \delta_i$. This relation will not be assumed to be symmetric, i.e. δ_i is not necessarily equal to δ_j, since it is conceivable that different robots may have different sensing capabilities This sensory model is similar to the *formation graph* network model proposed in [18]. Even though the thresholding model is somewhat crude, i.e. it does not model actual sensor capabilities exactly, it is still a rich enough model to capture some aspects that are relevant for illustrating the limitations they impose on any global coordination strategy, such as the one proposed in Section 2.

3.1 Global Formation Keeping

From (11) we directly see that as long as $\|\mathbf{x}_i(0) - \mathbf{x}_j(0)\| \leq \epsilon_{ij}$ for some $\epsilon_{ij} > 0$ then we can in fact guarantee that $\|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| \leq \min\{\delta_i, \delta_j\}$, $\forall t \geq 0$, as long as a_{ij} is sufficiently smaller than both δ_i and δ_j. The reason for this is as follows: Assume that we have $F(\mathbf{x}(0)) = 2\tau_{ij}(\epsilon_{ij}^2 - a_{ij}^2)^2$, i.e. that all other robots are in perfect formation. Since F is decreasing along trajectories, we must in fact have that

$$(\|\mathbf{x}_i(t) - \mathbf{x}_j(t)\|^2 - a_{ij}^2)^2 \leq (\epsilon_{ij}^2 - a_{ij}^2)^2. \quad (13)$$

If we assume that $\epsilon_{ij} > a_{ij}$ as well as $\epsilon_{ij} < \min\{\delta_i, \delta_j\}$ this directly implies that $\|\mathbf{x}_i(t) - \mathbf{x}_j(t)\| < \min\{\delta_i, \delta_j\}$ as well. This is furthermore true for all robot pairs. Thus, if the individual robots start close enough to each other, and the desired distance is achievable using the limited range sensors, then global formation maintenance can be achieved. An example of this is shown in Figure 2 (a).

3.2 Connected and Disconnected Components

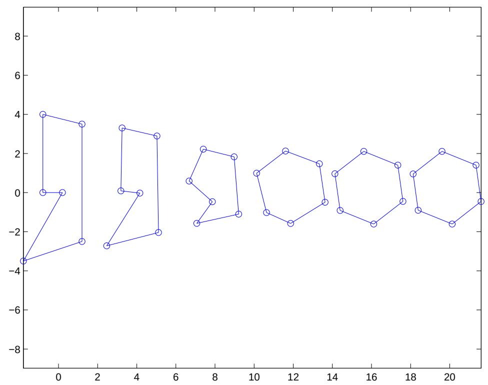
The assumptions in Section 3.1 that imply global formation keeping are clearly not always satisfied. In fact, if two robots are initialized outside each other's sensory range then Theorem 2.1 has no chance of holding true.

In [18] the notion of a *formation graph* was introduced. The idea is that each robot can be defined as a node in a graph, where the existence of an edge e_{ij} between nodes v_i and v_j implies that information can be shared between the two robots i and j. In [18] this graph structure is envisioned to capture the information that can be shared between different robots over a wireless network, but it can easily be modified to capture the limited range sensory situation in this paper. Since the range sensor limitations are potentially asymmetric (i.e. $\delta_i \neq \delta_j$), the resulting formation graphs would be *digraphs*. In other words, the edges are directed, and the digraph consists of a finite set V of nodes that correspond to the individual robots, and a collections of ordered pairs of disctinctive nodes from V. Any such pair is called a directred graph, denoted by $v_i v_j$, and the existence of such an edge imples that robot j is within robot i's sensory range. Such a digraph is strongly connected if every two nodes are mutully reachable from each other, but, due to the potential asymmetries in the range sensors ($\delta_i \neq \delta_j$) we focus our attention on weak connectedness instead. A semipath between v_i and v_j is an alternating seuqence of nodes and edges through the digraph from vi to v_j, and the digraph is said to be *weakly connected* if every two nodes are joined by a semipath, as defined in [4].

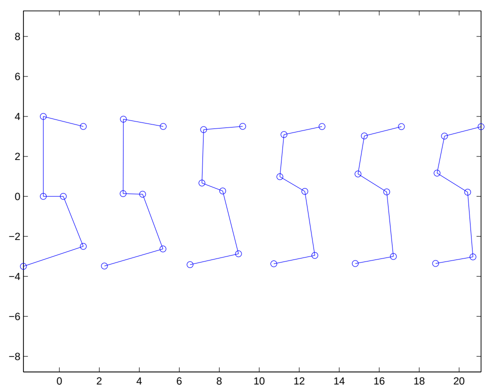
As seen in Figures 2 (b) and (c) there are different initial conditions in which the resulting formations are weakly connected and disconnected, and a conclusion to be drawn from these figures is that global formation keeping is not always possible. This implies that we need to study and design the local interactions between the different robots, i.e. shape the *social potential fields* [14, 20, 21] in such a way that limited range sensor capabilities can be modeled in a natural way.

3.3 Local Interactions

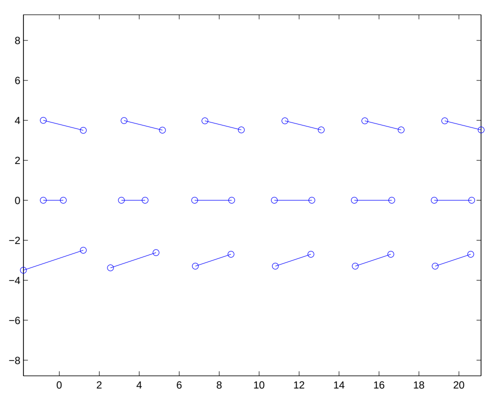
Since the direct approach to multi-agent coordination (i.e. to first define global formations and then try to achieve them) run into difficulties under limited range sensor capabilities, it seems natural to complement the direct approach with an inverse approach. In other words, design the local interactions first and then try to extract the global formation. A lot of work has gone in to the development of such social potential fields [14, 20, 21] and stability of the swarm dynamics has been shown for a number of different type of constructions [11] under the assumption that the robots can observe each other perfectly. The main idea is to define an attractive potential field that draws the robots together, and to augment it with a short-range repelling component in order to avoid collisions. However, it is straight forward to reshape these potential fields in such a way that they cut off at a given thresholding value, which would then incorporate the limited range sensor capabilities. To formally show what global formations would come out of this constructions has not yet been done, and it is an interesting avenue of research that will be pursued in the future.



(a)



(b)



(c)

Figure 2: In the top figure the initial conditions are such that the hexagonal formation is globally achievable. In the middle figure a weakly connected formation graph is obtained, while a disconnected formation graph is depited in the lower figure.

4 Conclusions

In this paper, we propose a model independent coordination strategy for multi-agent formation control. The problem is defined by a formation constraint in combination with a desired reference path for a non-physical, so called virtual leader. We show that if the robots track their respective reference points perfectly, or if the tracking errors are bounded, our method stabilizes the formation error. This is a very useful fact since it allows us to decouple the coordination problem into one planning problem, with proven features as long as the tracking is good enough, and one tracking problem.

However, we also show that by modeling the limited range sensor capabilities of the different robots as a thresholding function, global formation keeping can not always be achieved. In fact, situations can arise when the resulting formation, defined as a directed formation graph, is disconnected. What this implies is that unless perfect information is available, global formation maintenance can not always be achieved. This direct approach to solving the coordination problem (given a formation, define the individual motions) should thus be complemented with an inverse approach (given the local interactions, generate the global formation) for understanding the effect of limited sensory capabilities on the maltiagent coordination problem.

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