

Topology and Complexity of Formations

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Abstract

Biological multi-agent systems such as animal herds, insect colonies and fish schools provide a lot of insight into the study and design of artificial multi-agent systems such as teams of autonomous mobile robots. Similarly, a lot can be learned about biological systems by borrowing design and analysis tools from multi-agent robotics. In this paper some recent work in the area of multi-agent robotics by the authors has been summarized, which addresses some basic issues in the modelling of formations with limited sensory and communication capabilities. The basic idea is to model spatial relationships between agents as connectivity graphs. An information theoretic complexity measure of multi-agent formations is suggested, which is based on the complexity of connectivity graphs. The complexity measure helps find graphs and formations of the highest and the lowest complexities.

Keywords: Cooperative Control, Multi-Agent Robotic Systems.

1 Introduction

The interest in control and coordination of multi-agent robot teams has experienced a dramatic increase during the last few years (Egerstedt & Hu, 2001; Gazi & Pasino, 2003; Jadbabaie, Lin & Morse, 2003; Klavins, 2002; Muhammad & Egerstedt, 2003b; Ogren, Fiorelli & Leonard, 2002; Saber & Murray, 2001). Some of the techniques developed for single agents, interacting with both structured and unstructured environments, such as trajectory tracking, nonlinear control, mapping and localization, are readily applicable in the multi-agent case as well. However, a number of challenges, stemming from the distributed and hence local nature of the information available to the individual agents in the formation, have presented themselves. In order to look for inspiration when trying to model such systems, roboticists have increasingly begun to look to naturally occurring systems, where distributed, multi-agent systems are abundant. These systems range from human societies, where each agent is an extremely complex system in itself, and the social behavior transcends beyond simple mechanical tasks, to lifeless physical systems made of agents like particles, atoms or molecules. The latter carry no intelligence themselves, but interact using simple physical laws, and give rise to complex adaptive systems as a group. Robotics can be characterized as finding its place somewhere in between these two extremes. Needless to say, the comprehension of the complicated behavior of human societies may be the ultimate goal for a multi-agent system designer, but it is far too difficult and exists only in science fiction. The state of the art in multi-agent robotics is instead tackling significantly more humble objectives, like terrain exploration, coordinated building and manipulation, planning of team formations etc (Axelsson, Muhammad & Egerstedt, 2002; Balch & Arkin, 1998; Beard, Lawton & Hadaegh, 2001; Fierro et al., 2001; Lawton, Beard & Young, 2000;

Mataric, Nilsson & Simsarian, 1995; Reif & Wang, 1999; Tanner, Pappas & Kumar, 2002). On the other hand, inspiration from lifeless physical systems is inadequate, as robotic platforms available today carry so much computing power, sensors and communication capabilities that they are capable of much more than the imitation of simple “inverse-square” laws.

Group behavior is manifested in various biological systems as an impressive result of organic evolution. Examples can be found among social insects, animal herds, bacterial colonies, schools of fish, formations of flying birds, and so on. These group behaviors are very similar to the ones exhibited in multi-agent robotics, because of the following reasons:

1. *Local Interactions*: Individuals in animal groups interact only locally with their immediate neighbors and in many cases there is an absence of leader-hierarchy. This model is similar to fully decentralized, artificial multi-agent systems made up of identical members with limited individual sensor ranges. The emergence of a rich and complex global behavior from local interactions is of prime interest in multi-agent robotics.
2. *Simple Individual Behavior*: In robotics, individual agents exhibit a relatively small number of simple interactions, which give rise to complex group behaviors (Reynolds, 1987). This model is relevant when studying animal groups in which the individuals interact in a small number of simple ways.
3. *Communications*: Communication is an essential part of coordination in animal groups and the physical methods of exchanging information has a rich variety. In insects, for example, they communicate for alarm and assembly, recruitment, recognition, signalling presence of food, grooming and a host of other activities (Wilson, 1971; Bonabeau et al., 1997). Individual robots can also be equipped with communication channels for coordination. The issues concerning suitable information exchange for coordination are still open and active research problems.
4. *Group size, Complexity, and Randomness*: In insect societies, it has been observed that individual behavior is influenced by the colony size (Anderson & Ratnieks, 1999; Wilson, 1971). In larger societies, individual workers tend to make their decisions by collecting advice from their neighbors, and performing some kind of averaging or voting. In smaller colonies, the worker relies more on its own judgement than information exchanged from other workers. This explains why the insect’s actions may look erratic and seemingly random at the individual level, but give rise to order on a global level and why bigger colonies seem more ordered. These issues are related to design criteria in multi-agent systems, like how small should the tracking errors of individual control laws be, or what should be the resolution of the sensors, and finally how are these individual design factors related to the team size?

It can therefore be concluded that there is a remarkable similarity between the group behaviors found in biological systems and the ones roboticists want their artificial systems to exhibit. Similarly, a lot can be learnt from the abstract artificial robotic systems when modelling behaviors for animal groups. In this paper we present graph-theoretic models that are helpful to study the complexity and topology of formations in robots that interact locally with their neighbors. From the discussion above, it can be expected that these models might prove useful for the study of animal groups and social insects, as well as for understanding the coordination of multiple mobile robots.

2 Models of Formations

In many multi-agent systems, such as insect societies, animal herds, and robot teams, the individual agents can collect information about their environment and other agents by either peer to peer communication or by relying on sensory information. Since any physical sensor is always limited by its range and resolution, or by calibration errors, the information available to each agent by direct observation (or state estimation) is always limited and uncertain. Sensory limitations may also arise due to directivity patterns of sensors, e.g. the conic field of view of an eye or a camera, the radiation patterns of wireless antennas, sonars and lasers in robots.

Similarly, if we let the agents share information using peer to peer communication strategies, the possibility to convey and use global information is limited due to bandwidth limitations, weaker reception at large spatial distances, or the absence of feasible communication channels. This problem worsens as the formation size increases, both in cardinality and spatial dimension. Hence no individual agent can be assumed to have complete knowledge about the states of every other agent. This limitation directly leads to the question about how the local interactions should be represented. An obvious choice is to let the existence of such interactions be represented by edges in graph-based models (Muhammad & Egerstedt, 2003a; Jadbabaie, Lin & Morse A, 2003; Saber & Murray, 2003; Tanner, Pappas & Kumar, 2002).

A natural way to model the limitations of interaction among agents is to define their *regions of influence*. A region is defined according to the sensory range of an agent or the maximum distance by which it can communicate with other agents. For robotics applications, this makes perfect sense, but this also holds for biological multi-agent systems like social insects and fish schools, in which agents only interact with their neighboring agents. Therefore, it is interesting to study the class of graphs, based on a limited regions of influence. In a recent work by Muhammad & Egerstedt (2003b), the case was investigated when all agents live in a two dimensional Euclidean space, and have similar circular regions of influence of radius δ centered at their positions. The situation is similar to Figure 1, where ant 1, cannot interact with ants 5 and 6, because they are outside its region of influence, but it can interact with ants 2, 3, and 4. A graph can now be constructed, where nodes correspond to agents, and there exists an edge between two agents if the one agent lies in another's region of influence. These graphs have been named as *connectivity graphs* by the authors (Muhammad & Egerstedt, 2003b). The space of all connectivity graphs on N agents is denoted as $\mathcal{G}_{N,\delta} \subseteq \mathcal{G}_N$, where \mathcal{G}_N is the space of all possible graphs on N vertices. It can be immediately see that:

- The connectivity graphs are *simple* by construction i.e. there are no loops or parallel edges.
- They are undirected because all agents have the same radius for their regions of influence.
- The motion of individual agents in the formation may result in the removal or addition of edges in the connectivity graph, and therefore the graph is a dynamic structure.
- Every graph is not a connectivity graph.

An arbitrary graph exists as a connectivity graph if it has a valid *realization* in the configuration space of agents. Many realizations can correspond to the same graph. Although, this can be stated more rigorously, as given by Muhammad & Egerstedt, (2003b), the basic idea is straightforward. There are many interesting examples of realizable and non-realizable connectivity graphs. If a graph is completely disconnected, it means that the distance between any two agents in the formation

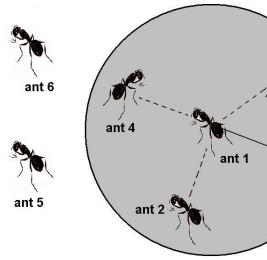


Figure 1: Agents and their region of influence

are separated by more than the distance δ . This can easily be achieved by placing each agent in such a way that it lies outside the regions of influence of all other agents. Therefore all completely disconnected graphs are realizable as connectivity graphs. If a graph has many disjoint connected components, each connected component can be placed “far away” from all other components so that none of the agents in one component lie in the regions of influence of agents in other connected component. By this construction, a realization for this graph can be obtained if and only if all components are realizable individually. Similarly, complete graphs, where an edge exist between all nodes can easily be produced, if the agents are placed very close to each other. Therefore, the study of realizability of graphs can be confined to connected graphs only. Using this technique one can now ask the question: when do graphs *not* exist as connectivity graphs?. Muhammad & Egerstedt (2003b), proved the following theorem.

Theorem 2.1 *The space of connectivity graphs over N agents $\mathcal{G}_{N,\delta}$, is a proper subset of the space of all possible graphs over N vertices \mathcal{G}_N , if and only if $N \geq 5$.*

The proof involves giving examples of graphs that cannot be realized for $N \geq 5$. Examples of non-realizable graphs for 5 and 6 vertices are shown in Figure 2. That these graphs are not realizable, can be seen from geometrical arguments. In fact, the “star” graphs of the type given for $N = 6$, do not exist for all $N \geq 6$, which completes the proof. This theorem helps understand that *not all graphs are valid models for multi-agent formations.*

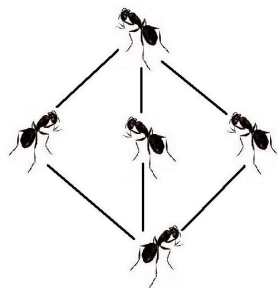


Figure 2: Graphs that are not connectivity graphs.

The importance of this characterization of the space of connectivity graphs can also be understood as follows. From the discussion above, it can be seen that totally disconnected graphs and totally connected (complete) graphs are trivially realizable. However these two extremes are not very interesting from a behavioral point of view. The disconnected case corresponds to the situation where there is no interaction between agents. In the completely connected case however, the agents are packed so tightly that the system becomes fully coupled and the global information is available at all agents. The central theme of multi-agent coordination i.e. global behavior from local rules therefore becomes irrelevant. Hence the situation of perhaps the greatest interest is between the two extremes when the graph is not necessarily complete or even connected, and when no strictly proper subset of the graph's vertices is isolated from the rest. It is precisely this class of graphs and their respective realizations, that give rise to the rich variety of global behaviors from simple local rules.

There are several results that have been proven by the authors about connectivity graphs. In another work by Muhammad & Egerstedt (2003a), it has been shown how to obtain subgraphs of connectivity graphs that resemble simplicial complexes, which are used in algebraic topology to distinguish between different "shapes". However the most interesting results, from the point of view of biological multi-agent systems, describe the complexity of multi-agent systems in terms of the complexity of their connectivity graphs. The results of this study has been a motivation for designing algorithms to produce low-complexity multi-robot formations called δ -chains, that need a small number of interactions to maintain formation. The work on complexity of multi-agent systems is summarized below followed by some observations on its relevance to biological multi-agent systems.

3 Complexity of Multi-agent Systems

It has recently been shown by Muhammad & Egerstedt (2003c) that the type of graphs called δ -chains have the lowest complexity among all multi-agent formations. A δ -chain is a connected graph, which is also a Hamiltonian path on all nodes. See Figure 3. If X_j represent the state associated with an agent $1 \leq j \leq N$, the *intrinsic structural complexity* of the multi-agent formation is defined as:

$$C(\mathcal{F}) = \sum_j \sum_{i \neq j} F_{i,j}(X_j), \quad (1)$$

where each $F_{i,j}$ is the *information flow* at agent j due to agent i according to some given communication protocol. The information flow at an agent, is the time rate of information exchange taking place at that agent due to either or both sensory perception or communication. It was also shown that the two modes of information exchange are equivalent from an information theoretic point of view. Also, the presence of protocols implies that every interaction is not active during a certain time period. Therefore the intrinsic complexity is bounded above by a quantity that assumes that all interactions are active for all time. This bound is in-fact a complexity associated with a *broadcast protocol*.

If Δt is the minimum permissible time for information exchange in the system (due to either bandwidth, sensor update interval, or algorithm execution cycle), then it can be seen that protocols of synchronous information exchange, which are more complicated than the broadcast protocol, would result in a decrease of the total information flow. Let us denote a formation as $\mathcal{F} = (X_1, X_2, \dots, X_N)$, and denote the complexity of a formation, associated with the broadcast protocol as $C_B(\mathcal{F})$, then

$$C_B(\mathcal{F}) \geq C_P(\mathcal{F}),$$

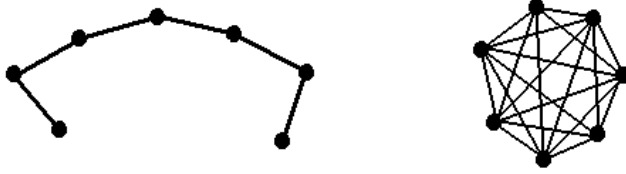


Figure 3: δ -chain and complete graph for 7 vertices.

where $C_P(\mathcal{F})$ is the complexity for some arbitrary protocol. $C_B(\mathcal{F})$ is therefore the worst case complexity associated with a particular formation. The information flow of a remote state X_j at agent i , according to this protocol, is

$$F_{i,j}(X_j) = \frac{I(X_j; \mathcal{Z}_{j,i})}{k_{ij}\Delta t}, \quad (2)$$

where $\mathcal{Z}_{j,i}$ is the sensory measurement of the sensor on board agent j , (or the equivalent virtual sensor for the communication channel between agents i and j), and $I(X_j; \mathcal{Z}_{j,i})$ is the information obtained about X_j from $\mathcal{Z}_{j,i}$ (Cover & Thomas, 1991), and k_{ij} is an integral multiple of Δt so that $k_{ij}\Delta t$ is the sensor update time.

We also showed that the complexity $C_B(\mathcal{F})$ is bounded above as

$$C_B(\mathcal{F}) \leq \sum_j \sum_{i \neq j} \text{deg}(v_j) \frac{I(X_j; \mathcal{Z}_{j,i})}{k_{ij}\Delta t},$$

where $\text{deg}(v_i)$ is the number of agents being sensed (or communicated with directly) by agent i . Furthermore, if the states exchanged by all agents are of the same type and encoded in the same way, $I(X_j; \mathcal{Z}_{i,j}) = \gamma$, then we can write

$$C_B(\mathcal{F}) \leq \frac{\gamma}{\Delta t} \sum_i \left(\text{deg}(v_i) + \sum_{v_j \notin \text{star}(v_i)} \frac{\text{deg}(v_j)}{k_{ij}} \right).$$

Compare this to the complexity defined on a graph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$, in the context of molecular chemistry (Plavsic & Plavsic, 2002).

$$C(G) = \sum_{v_i \in \mathcal{V}} \left(\text{deg}(v_i) + \sum_{v_j \in \mathcal{V}, v_i \neq v_j} \frac{\text{deg}(v_j)}{d(v_i, v_j)} \right), \quad (3)$$

where $d: \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}^+$ is some distance function defined between vertices.

Therefore it is easy to see that,

$$C_B(\mathcal{F}) \leq \frac{\gamma}{\Delta t} C(G),$$

where G is the connectivity graph of the formation. This relationship leads to the following observation. *The complexity of the connectivity graph of a formation is a (tight) upper bound for the worst case complexity associated with an arbitrary protocol of communication in a multi-agent formation.*



Figure 4: δ -chains and V-formations in bird flight. (Copied with permission from A. Filippone, UMIST, UK).

Therefore the study of structural complexity of multi-agent formations is closely related to the complexity of their connectivity graphs. With these considerations in mind, the following theorem was proved by Muhammad & Egerstedt (2003c).

Theorem 3.1 *If G is a connected connectivity graph on N vertices, then the complexity of the graph G is bounded above and below as*

$$C(\delta_N) \leq C(G) \leq C(\mathcal{K}_N), \quad (4)$$

where δ_N is the δ -chain and \mathcal{K}_N the complete graph on N vertices.

This theorem gives the justification for studying δ -chains as low-complexity formations. The δ -chains are interesting objects in the context of biological multi-agent systems. These chains are examples of formations that can be maintained with minimum coordination. One of the most interesting manifestations of these chains can perhaps be seen in formation flight of birds, specially the V-formations. See Figure 4. Although, there have recently been studies that relate this type of formation flight to energy conservation (Weimerskirch et al., 2001), nevertheless the aspect of minimum coordination is hard to overlook in this case. In other naturally occurring multi-agent systems, δ -chains can be observed in queues, lines, caravans and flanks, that can be maintained with minimum interaction between agents. This gives an additional leverage to the complexity definition of Equation 1, which has also been shown to have a remarkable similarity with complexity measures for chemical graphs (Muhammad & Egerstedt, 2003). Therefore these complexity measures may be useful in comparing formations of various natural and artificial multi-agent systems.

4 Conclusions

The introduction of connectivity graphs for characterizing the local interactions in multi-agent formations serves two purposes. First, since these interactions imply constraints on the movements of the individual agents, it is vitally important that the set of feasible formations can be characterized in a precise manner. This has been described as the space of all connectivity graphs for a fixed number of agents. Secondly, and perhaps more importantly, these graphs provide guidance as to how the information should flow between different agents in order for the team of agents to come

up with plans for achieving global objectives in a decentralized manner. Therefore, these graph theoretic models help us to study important aspects in the topology, complexity and coordination of multi-agent systems. This abstraction of multi-agent systems makes it possible to compare and relate behaviors in natural and artificial multi-agent systems and may provide useful in strengthening this connection, in addition to its original objective of advancing techniques in design and implementation of autonomous robot teams.

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