AUTONOMOUS FORMATION SWITCHING FOR MULTIPLE, MOBILE ROBOTS

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Abstract: In this paper we investigate the question concerning what multi-agent formations to use in a given situation. In particular, we show how it is possible to produce a control strategy for teams of mobile robots that switches between different formations as a reaction to environmental changes. The feasibility of the approach is verified in simulation, where a steepest descent algorithm is combined with standard reactive behaviors that ensure that the individual robots avoid neighboring obstacles and robots, while approaching a desired target location.

Keywords: Mobile robots, coordinated control, hybrid systems

1. INTRODUCTION

The problem of controlling multiple, mobile robots in a coordinated fashion, i.e. to enforce desired formations, has received considerable attention during the last decade. Of particular concern has been the development of centralized or decentralized control laws that guarantee formation stability (Egerstedt and Hu, 2001; Gazi and Passino, 2002; Lawton *et al.*, 2000; Swaroop and Hedrick, 1996; Ögren *et al.*, 2002; Beard *et al.*, 2001; Fierro *et al.*, 2001). Recent work has furthermore focused on information topology, i.e. a study of what information the individual robots need to communicate in order to maintain formation (Egerstedt *et al.*, 2002; Olfati-Saber and Murray, 2002*a*; Tanner *et al.*, 2002; Eren *et al.*, 2002; Fax and Murray, 2002). The underlying driver of these research efforts is the implicit assumption that there is strength in numbers, which has been exploited when exploring and negotiating unknown or hostile environments (Balch and Arkin, 1998; Mataric *et al.*, 1995).

However, a fundamental question in multi-agent robotics that has been somewhat neglected concerns what formations to use in a given situation. For example, in (Desai *et al.*, 2001) it is pointed out that the robots should be spread out when navigating and exploring free-space, while a more tight formation is to prefer when negotiating cluttered environments.

In this paper we make this type of observation concrete by proposing a *hybrid* control architecture in which the robot team switches between different formations as a reaction to environmental changes. Previous work in this area has assumed an *a priori* sequence of formations to be used, with known switch times (Desai *et al.*, 2001). Alternatively, theoretically elegant yet computationally cumbersome game theoretic switch-laws have been generated from the Hamilton-Jacobi-Bellman equation (Lygeros *et al.*, 1999). The main contribution in this paper is thus the development of a computationally efficient, hybrid control law in which the discrete transitions occur *au-*

tonomously. To this end, we define a *formation error* with respect to each possible formation under consideration, and let the robot team execute the formation with the smallest error. This is achieved by combining a negative gradient method for formation maintenance (as suggested in (Egerstedt and Hu, 2001; Egerstedt *et al.*, 2002)) with a set of *reactive behaviors* (see for example (Arkin, 1998)) that ensure safety with respect to obstacles as well as neighboring robots.

In this paper we model the formation switching using a *hybrid automaton*, in which each discrete node corresponds to a particular formation. In Section 2 we introduce the formation errors and show how they can be used for generating stable motions by letting the individual robots move in a negative gradient direction. In Section 3 we combine these maneuvers with standard, reactive behaviors. The formation switching is presented in Section 4, followed by a brief discussion about what implications limited sensory range constraints have on the proposed control strategy, in Section 5.

2. ROBOT FORMATIONS

In order to generate a hybrid control strategy in which the formation associated with the smallest error is enforced, it is crucial that a quantitative description can be given that characterizes how well a given formation is maintained. If we assume that each individual robot is evolving according to a controlled differential equation

$$\begin{aligned} \dot{z}_i &= f_i(z_i, u_i) \\ x_i &= g_i(z_i), \ i = 1, \dots, N, \end{aligned}$$

where $z_i \in Z_i$ and $u_i \in U_i$ denote the states and inputs associated with robot i, and N is the total number of robots in the team. Furthermore, $x_i \in \mathbb{R}^n$ is an output function that maps the state of robot i to $\mathbb{R}^n$, which enables us to define the *formation error* as a mapping $V : \mathbb{R}^{N\times n} \to \mathbb{R}_+ \cup \{0\}$.

Throughout this paper we will study planar robots whose dynamics are of the unicycle-type

$$\begin{aligned} \dot{z}_{i1} &= u_{i1}\cos(z_{i3}) \\ \dot{z}_{i2} &= u_{i1}\sin(z_{i3}) \\ \dot{z}_{i3} &= u_{i2}, \end{aligned}$$

where $(z_{i1}, z_{i2}) \in \mathbb{R}^2$ is the position of robot i, $z_{i3} \in S^1$ is the orientation of the robot, and the control inputs (u_{i1}, u_{i2}) represent the translational and angular velocities respectively. If we were to let the formation error be defined with respect to the position of the robots, we would get

$$\begin{pmatrix} x_{i1} \\ x_{i2} \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \end{pmatrix} \begin{pmatrix} z_{i1} \\ z_{i2} \\ z_{i3} \end{pmatrix}.$$

In this paper we furthermore choose to define the formation errors in terms of the Euclidean distances between the positions of the different agents in the team. This restriction is not severe since it captures the vast majority of the different formations that have been proposed (Olfati-Saber and Murray, 2002*a*; Reif and Wang, 1999). In order to characterize a particular formation we can thus proceed as follows: We let $\Delta_{ij} = \Delta_{ji}$ be the desired distance between robots i and j, and set

$$V(x_1, \dots, x_N) = \sum_{i=1}^{N} \sum_{j\neq i} \tau_{ij} \left(\|x_i - x_j\|^2 - \Delta_{ij}^2 \right)^2,$$

where $\tau_{ij} = \tau_{ji} \geq 0$ is a weight that corresponds to the relative importance of enforcing the correct distance between robots i and j. This construction ensures that V is positive semi-definite as well as $V(x_1, \dots, x_N) = 0$ only if the desired formation is perfectly achieved (with respect to the x_i's), i.e. when $\|x_i - x_j\| = \Delta_{ij}$. Moreover, it should be noted that the kernel of V is in general not unique, i.e. it contains more than one point, which corresponds to a purposeful under-determination of the formation. For example, if $\tau_{ij} > 0, \ \forall i, j$ (and if $\text{Ker}(V) \neq \emptyset$), we have specified the formation up to a rotation of the entire formation.

Now, given a formation V, we need to specify the evolution of the individual robots in such a way that V is reduced to zero. For planar unicycles it is well known that no smooth, constant feedback can stabilize the system, but, as observed in (Olfati-Saber and Murray, 2002*b*), we can easily stabilize a point $(\tilde{z}_{i1}, \tilde{z}_{i2})$ that lies a distance l along the line that is normal to the wheel axis, using feedback linearization techniques. In fact, let $\tilde{z}_{i1} = z_{i1} + l\cos(z_{i3}), \ \tilde{z}_{i2} = z_{i2} + l\sin(z_{i3})$. This directly gives that $\dot{\tilde{z}}_{ij} = \tilde{u}_{ij}, \ j = 1, 2$, with

$$\begin{pmatrix} \tilde{u}_{i1} \\ \tilde{u}_{i2} \end{pmatrix} = \begin{pmatrix} \cos(z_{i3}) & -l\sin(z_{i3}) \\ \sin(z_{i3}) & l\cos(z_{i3}) \end{pmatrix} \begin{pmatrix} u_{i1} \\ u_{i2} \end{pmatrix}.$$

The new output function $(x_{i1}, x_{i2}) = (\tilde{z}_{i1}, \tilde{z}_{i2})$ then gives us a system whose dynamics is given by two single integrators. For such a system, we can set

$$\dot{x}_i = -\gamma \frac{\partial V(\mathbf{x})}{\partial x_i},$$

where $\gamma > 0, \ \mathbf{x} = (x_1, \dots, x_N)^T$, and where

$$\frac{\partial V(\mathbf{x})}{\partial x_i} = 8 \sum_{j\neq i} \tau_{ij} \left(\|x_i - x_j\|^2 - \Delta_{ij}^2 \right)(x_i - x_j).$$

Using such a gradient descent directly gives that

$$\frac{dV(\mathbf{x})}{dt} = \left(\frac{\partial V(\mathbf{x})}{\partial \mathbf{x}} \right)^T \dot{\mathbf{x}} = -\gamma \left\| \frac{\partial V(\mathbf{x})}{\partial \mathbf{x}} \right\|^2,$$

which is negative as long as $\mathbf{x} \notin \text{Ker}(V)$, i.e. this control law stabilizes the formation error.

An example of applying this control strategy to the case of three robots specified to move in a tri-

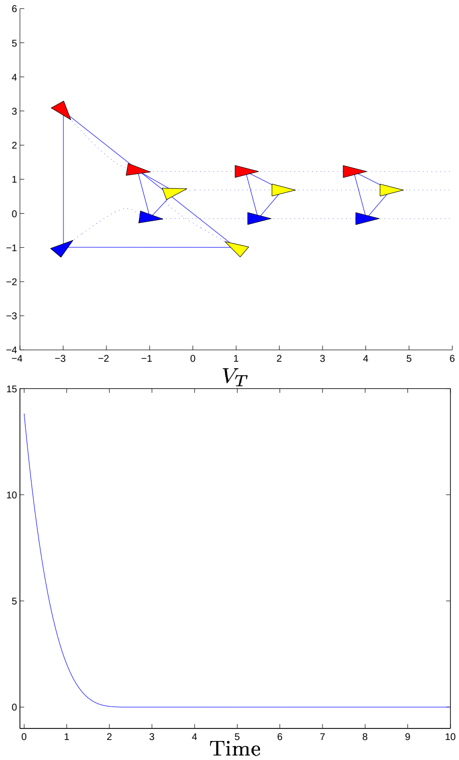

Fig. 1. Three robots converging to an isosceles triangle ($\Delta_{12} = 1$, $\Delta_{13} = \sqrt{2}$, $\Delta_{23} = 1$) and the corresponding error-function V_T.

angular formation is shown in Figure 1. Depicted is also the corresponding error function for the formation $V_T(x_1, x_2, x_3)$, where the subscript T denotes a triangular formation.

3. BEHAVIOR-BASED ROBOTICS

So far we have proposed a steepest descent motion for the individual robots that guarantees that they will stabilize a given formation, i.e. that they will drive the formation error to zero asymptotically. However, using this control strategy alone, we have no guarantees that the robots will not collide with each other, stay away from obstacles, or approach a desired target. For this, a standard solution is to construct a collection of behaviors, i.e. a collection of controllers dedicated to performing particular tasks such as avoid-obstacles or approach-target.

Consider robot i where we, as before, use (z_{i1}, z_{i2}) to denote the position of the robot, and z_{i3} to denote orientation. If we assume that the robot can measure the distances and angles to neighboring obstacles and robots, we can define a behavior as a mapping from state and sensory data to the control inputs. A standard way (Arkin, 1998) of specifying the effect of individual behaviors is to let the behavior define a vector

$$\mathcal{B} = r_{\mathcal{B}} \begin{pmatrix} \cos(\phi_{\mathcal{B}}) \\ \sin(\phi_{\mathcal{B}}) \end{pmatrix},$$

where $r_{\mathcal{B}}$ is the magnitude of the behavior vector, and $\phi_{\mathcal{B}}$ is the orientation. This vector formalism allows us to directly generate appropriate control values, and in this paper we let

$$\begin{pmatrix} u_{i1} \\ u_{i2} \end{pmatrix} = F(\mathcal{B}) = \begin{pmatrix} \min\{v_0, 1/r_{\mathcal{B}}\} \\ C(\phi_{\mathcal{B}} - z_{i3}) \end{pmatrix}.$$

In other words, the translational velocity achieves its nominal value $v_0 > 0$ when the magnitude of the behavior vector is small, but is reduced as this magnitude grows. Furthermore, the angular velocity is simply given by a proportional error feedback law, with $C > 0$ being the gain.

Now, if we assume that $\mathcal{B}_1$ and $\mathcal{B}_2$ are vectors corresponding to two different behaviors, they can be combined directly using a vector addition operation $\mathcal{B}_1 + \mathcal{B}_2$ in order to produce a new behavior.

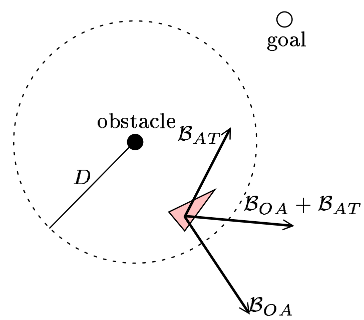

Fig. 2. Vector addition of obstacle-avoidance and approach-target behaviors.

Of particular importance to the development in this paper are the *obstacle-avoidance* (denoted OA) and *approach-target* (denoted AT) behaviors. Most mobile robots are equipped with a collection of k range-sensors, e.g. a standard sonar-ring typically consists of 8 or 16 ultra-sonic sensors. Each sensor measures the distance to the closest obstacle along a particular, fixed relative orientation. For robot i, we let d_{ij} denote the distance to the closest obstacle detected by sonar j, and we let ϕ_{ij} be the corresponding angle. We can then define the obstacle avoidance behavior $\mathcal{B}_{OA}$ through the behavior vectors $\mathcal{B}_{OA} = \mathcal{B}_{OA,1} + \mathcal{B}_{OA,2} + \ldots + \mathcal{B}_{OA,k}$, where

$$r_{\mathcal{B}_{OA,j}} = \begin{cases} 0 & \text{if } d_{ij} > D \\ C_{OA}\dfrac{(D - d_{ij})}{d_{ij}^2} & \text{otherwise} \end{cases}$$
$$\phi_{\mathcal{B}_{OA,j}} = \pi + \phi_{ij},$$

where $C_{OA} > 0$, and D is the safety distance at which the obstacle-avoidance behavior starts affecting the system, as seen in Figure 2.

In a similar manner, we can let $\mathcal{B}_{AT}$ be given by

$$r_{\mathcal{B}_{AT}} = C_{AT}$$
$$\phi_{\mathcal{B}_{AT}} = \text{atan}((x_{g2} - x_{i2})/(x_{g1} - x_{i1})),$$

where $C_{AT} > 0$, and the goal is located at (x_{g1}, x_{g2}), as seen in Figure 2.

Now, in order to complete the picture we need to include the negative descent direction as a formation behavior, and if we focus our attention on robot i, we let the corresponding behavior vector $\mathcal{B}_F$ be given by

$$r_{\mathcal{B}_F} = \gamma \left\| \frac{\partial V}{\partial x_i} \right\|$$
$$\phi_{\mathcal{B}_F} = \text{atan}\left((-\gamma \frac{\partial V}{\partial x_{i2}} - x_{i2}) / (-\gamma \frac{\partial V}{\partial x_{i1}} - x_{i1}) \right).$$

A simulation utilizing all of the three behaviors discussed above is shown in Figure 3. In that figure, three robots travel to a goal position (denoted by "*"), while maintaining a triangular formation and avoiding obstacles. Using all three behaviors simultaneously, the robots avoid the obstacles (represented by circles), and maintain the triangular formation until the robots reaches the "wall" of obstacles. After the robots have passed the "wall" they recover their triangle formation as the influence of the obstacle avoidance behavior decreases the further away from the wall the robots moves.

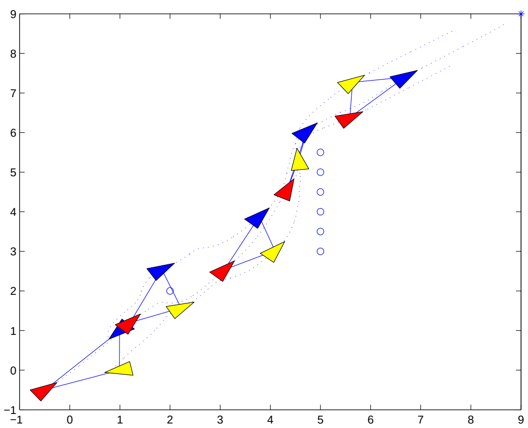

Fig. 3. Triangular formation negotiating obstacles.

4. FORMATION SWITCHING

Using the notation we have established in the previous section, we now want to allow for the possibility of letting the system switch between a collection of possible formations. To this end, we use $\mathcal{F}_j$ as shorthand for

$$\mathcal{F}_j = \begin{pmatrix} \cos(z_{i3}) & -l\sin(z_{i3}) \\ \sin(z_{i3}) & l\cos(z_{i3}) \end{pmatrix} F(\mathcal{B}_{OA} + \mathcal{B}_{AT} + \mathcal{B}_{Fj}),$$

where $\mathcal{B}_{Fj}$ is the behavior vector corresponding to formation j, and $F(\mathcal{B})$ is the behavior vector-to-input mapping given in the previous section. If the robot team is trying to achieve formation j, while avoiding obstacles, we then have $\dot{x}_i = \mathcal{F}_j$.

But, if we have a collection of potentially useful formations, with formation errors $V_1, V_2, \ldots, V_M$, and corresponding formation error functions $G_1(V_1)$, $G_2(V_2), \ldots, G_M(V_M)$, where the formation error functions are defined as mappings $G_i(V_i) : \mathbb{R} \mapsto \mathbb{R}$, we can define a transition relation $\mathcal{T}_j$ as $\mathcal{T}_j =$ TRUE iff $G_j(V_j(\mathbf{x})) \leq G_i(V_i(\mathbf{x}))$, $i = 1, \ldots, M$, such that we will always make a transition to the formation with the smallest $G_i(V_i(\mathbf{x}))$. We could of course let $G_i(V_i) = V_i$, but we want to allow for the possibility of scaling these functions, e.g. in order to capture the fact that a "spread-out" formation is preferable when no obstacles are close. The reason why we call $\mathcal{T}_j$ a *transition relation* is as follows: The dynamics of each robot i is given by $\dot{x}_i = \mathcal{F}_{j^\star}$, where

$$j^\star = \underset{j=1,\ldots,M}{\arg\min} \{G_j(V_j(\mathbf{x}))\},$$

i.e. the transition relations trigger transitions in the hybrid automaton $\mathcal{H}_i$ used for describing the motion of robot i, as shown in Figure 4.

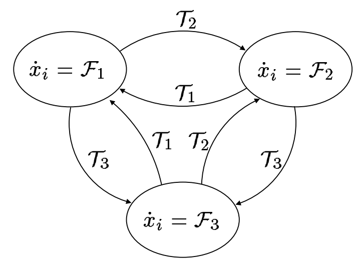

Fig. 4. A hybrid automaton representing three formations.

A simulation where we switch between a line and a triangular formation is shown in Figure 5, together with the corresponding error functions V_L and V_T respectively. Here we assume $G_i(V_i) = V_i$, and as the robots travel towards the goal (located at $(9, 9)$) they need to propagate through a narrow passage illustrated by the four objects located at $(2, 3), (3, 2), (4, 3)$ and $(3, 4)$. The obstacle avoidance behavior deforms the triangle so that $V_L < V_T$ and accordingly we make a transition to the line formation. After we have made the transition each robot's dynamics is given by $\dot{x}_i = \mathcal{F}_L$ and accordingly, the robots strive to be in a line formation. When the formation arrives at the obstacle located at $(5, 5)$ the obstacle avoidance behavior puts the leading robot out of its current heading resulting in $V_T < V_L$ and the robots make a transition back to the triangle formation and the dynamics of each robot is once again given by $\dot{x}_i = \mathcal{F}_T$.

5. LIMITED SENSORY RANGE CONSTRAINTS

The conclusions to be drawn from the previous sections is that it is possible to formulate a multi-agent coordination strategy in which the robot team switches between different formations in an autonomous fashion. However, for this strategy to be successful, $\frac{\partial V_j}{\partial x}$ and $G_j(V_j(\mathbf{x}))$ must be computable by the each individual robot.

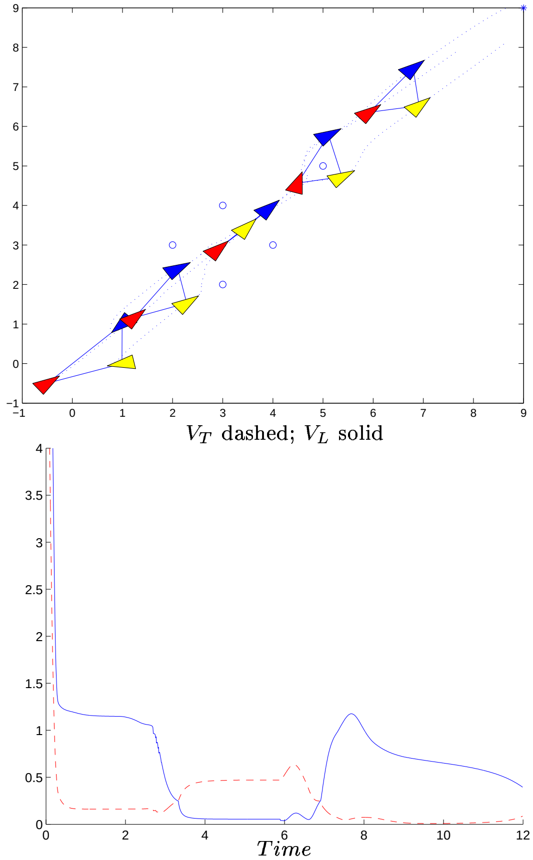

Fig. 5. Three robots switching between a line and a triangle formation and their corresponding error functions.

If we let the error function associated with formation j be given by

$$V_j(\mathbf{x}) = \sum_{i=1}^{N} \sum_{k \neq i} \tau_{ik} \left(\|x_i - x_k\|^2 - \Delta_{ik}^2 \right)^2$$

$$\triangleq \sum_{i=1}^{N} \sum_{k \neq i} \frac{W_{ik}(x_i, x_k)}{2},$$

we directly note that robot i can compute

$$\frac{\partial V_j(\mathbf{x})}{\partial x_i} = \sum_{k \neq i} \frac{\partial W_{ik}(x_i, x_k)}{\partial x_i}$$

as long as it has perfect information about all x_k's such that $\tau_{ik} > 0$. For the remainder of this section we thus investigate the performance of the gradient descent algorithm under the assumption that robot i can only measure its neighbors at a distance less than or equal to δ_i. The formation error computable by robot i becomes

$$V^i(\mathbf{x}) = \sum_{j \neq i \ | \ \|x_i - x_j\| \leq \delta_i} W_{ij}(x_i, x_j).$$

Previously, we used $\dot{x}_i = -\gamma \frac{\partial V}{\partial x_i}$, but we gain additional control freedom if we now let

$$\dot{x}_i = -\sum_{j \neq i} \gamma_{ij} \frac{\partial W_{ij}(x_i, x_j)}{\partial x_i},$$

where we simply let $\gamma_{ij} = 0$ if $\|x_i - x_j\| > \delta_i$.

The question then becomes that of picking the gains γ_{ij} (possibly as a function of $\mathbf{x}$) in such a way that the formation is stabilized even under limited information assumptions. We address this question by computing the temporal differences for the individual terms in the error function. We get that $\frac{\Delta W_{ij}(x_i, x_j)}{h}$ is given by

$$\left(\frac{\partial W_{ij}}{\partial x_i} \right)^T \dot{x}_i + \left(\frac{\partial W_{ij}}{\partial x_j} \right)^T \dot{x}_j + \mathcal{O}(h),$$

which can be reduced to

$$\frac{\Delta W_{ij}(x_i, x_j)}{h} = -(\gamma_{ij} + \gamma_{ji}) \left\| \frac{\partial W_{ij}}{\partial x_i} \right\|^2 \sum_{k \neq i,j} (\gamma_{ik} - \gamma_{jk}) \left\langle \frac{\partial W_{ij}}{\partial x_i}, \frac{\partial W_{ik}}{\partial x_i} + \frac{\partial W_{jk}}{\partial x_j} \right\rangle + \mathcal{O}(h)$$

based on the observation that

$$\frac{\partial W_{ij}}{\partial x_i} = -\frac{\partial W_{ij}}{\partial x_j}.$$

Note, this is true only if $\delta_1 = \delta_1 = \ldots = \delta_N$, which we assume to be the case. Now, it is sufficient to show that all the temporal differences $(\Delta W_{ij}/h)$ are negative in order to ensure that the robots stay within sensory range of each other while achieving the formation. We thus need to compute gains such that

$$(\gamma_{ij} + \gamma_{ji}) \left\| \frac{\partial W_{ij}}{\partial x_i} \right\|^2 - \sum_{k \neq i,j} (\gamma_{ik} - \gamma_{jk}) \left\langle \frac{\partial W_{ij}}{\partial x_i}, \frac{\partial W_{ik}}{\partial x_i} + \frac{\partial W_{jk}}{\partial x_j} \right\rangle < 0.$$

The general study of this topic remains a future endeavor, but as an example the problem of controlling three robots can be solved in a straightforward manner. Assume that the formation is specified by the three distances $\Delta_{12}, \Delta_{23}, \Delta_{31}$. The formation error thus becomes

$$W_{12}(x_1, x_2) + W_{23}(x_2, x_3) + W_{13}(x_1, x_3),$$

with the corresponding dynamics

$$\begin{aligned} \dot{x}_1 &= -\gamma_{12} \frac{\partial W_{12}}{\partial x_1} \\ \dot{x}_2 &= -\gamma_{21} \frac{\partial W_{12}}{\partial x_1} - \gamma_{23} \frac{\partial W_{23}}{\partial x_3} \\ \dot{x}_3 &= -\gamma_{31} \frac{\partial W_{23}}{\partial x_3}. \end{aligned}$$

Here, robot 3 and 1 cannot observe each other, so the negativity of the temporal difference $W_{13}(x_1, x_3)$ is no longer relevant. We only need $\Delta W_{12}/h < 0$ and $\Delta W_{12}/h < 0$. These negativity constraints can be made stricter by reducing them to a single constraint $\Delta W_{12}/h + \Delta W_{12}/h < 0$, from which we get

$$
\begin{aligned}
&(\gamma_{21} - \gamma_{12}) \left\| \frac{\partial W_{12}}{\partial x_1} \right\|^2 \\
&+ (\gamma_{23} + \gamma_{21}) \left\langle \frac{\partial W_{12}}{\partial x_2}, \frac{\partial W_{23}}{\partial x_2} \right\rangle \\
&+ (\gamma_{23} - \gamma_{32}) \left\| \frac{\partial W_{23}}{\partial x_2} \right\|^2 < 0,
\end{aligned}
$$

for which it is straight-forward to see that it is always possible to find a set of $\{\gamma_{12}, \gamma_{21}, \gamma_{23}, \gamma_{32}\}$ such that the constraint is satisfied.

6. CONCLUSIONS

In this paper we show how it is possible to produce a control strategy for teams of mobile robots that switches between different formations as a reaction to environmental changes. By defining an error function associated with each formation, the system dynamics is governed by the formation with the smallest such error (or with the smallest cost associated with the error). The feasibility of the approach is verified in simulation, where a steepest descent algorithm is combined with standard reactive behaviors that ensure that the individual robots avoid neighboring obstacles and robots, while approaching a desired target location. The resulting control strategy can be cast as a hybrid automaton for each individual robot, while the behavior of the team can be viewed as a synchronous automata product between the individual hybrid automata.

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