EE-562: Robot Motion Planning
Midterm Exam

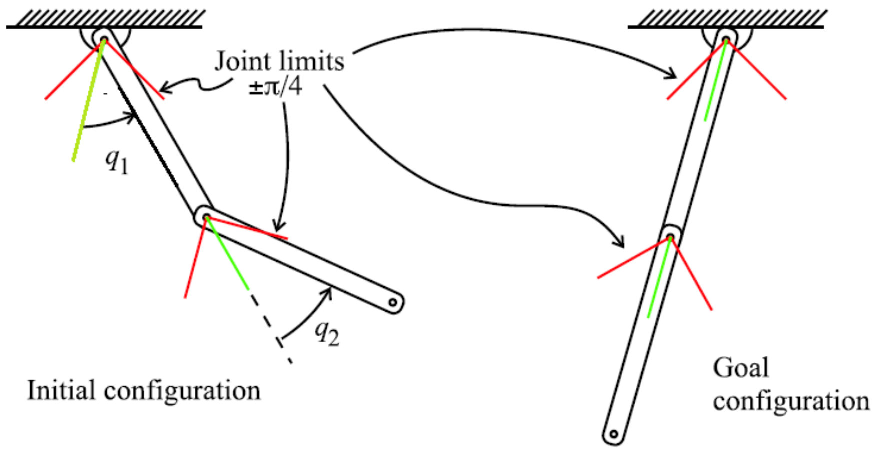
Name : ____________________________________

ID # : ____________________________________

Instructions.

1. Examination is *open book, open notes*.
2. Total points = 90.
3. All questions are compulsory. The point system may be used as a guideline to plan the time spent on each question.
4. LUMS honor-code applies on fair practices during exam.
5. Try to explain your solutions in clear unambiguous words or mathematical expressions. Marks will not be awarded on what you *intended* to write but what you actually wrote down.

Q 1	36 points	
Q 2	36 points	
Q 3	18 points	



Problem 1 [8+8+20 = 36 Points]

Consider a double pendulum as shown in the figure above.

1. Give the degrees of freedom, configuration space and topology of the system when the joints are free to rotate about their hinges.

2. Give the degrees of freedom, configuration space and topology of the system when the joints can only rotate upto $\pm\pi/4$ about the zero angles. (the limits are sketched in the figure about the zero axis)

3. Write down a navigation function for the system with joint restrictions with the goal at $(q_1, q_2) = (0, 0)$.

Hint. If you have a navigation function $\phi_1(q_1)$ for CSpace X_1 and another navigation function $\phi_2(q_2)$ for CSpace X_2, then we can set up a *joint* navigation function $\Phi(q_1, q_2)$ on $X_1 \times X_2$ by the formula

$$\Phi(q_1, q_2) = \sigma(k_1 \frac{\phi_1(q_1)}{1 - \phi_1(q_1)} + k_1 \frac{\phi_2(q_2)}{1 - \phi_2(q_2)}),$$

where $k_1, k_2 > 0$ are two arbitrary constants and $\sigma(.)$ is the analytical switch.

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Problem 2 [18+18 = 36 Points]

1. Setup a potential function on a bounded grid shown below using the wavefront planner. The robot can move to all eight unoccupied neighbors of its current cell. The obstacles are marked by boxes ■ and the goal is represented by the symbol ⊛ in coordinate $(2, 5)$. Finally show a trajectory of the robot from an initial position $(1, 1)$ in the upper left corner to the goal. The grid is repeated twice to allow rough work or to present a clear final solution.

	1	2	3	4	5	6
1		■	■			
2			■	■	⊛	
3				■		
4			■			
5						■
6					■	
7	■	■				

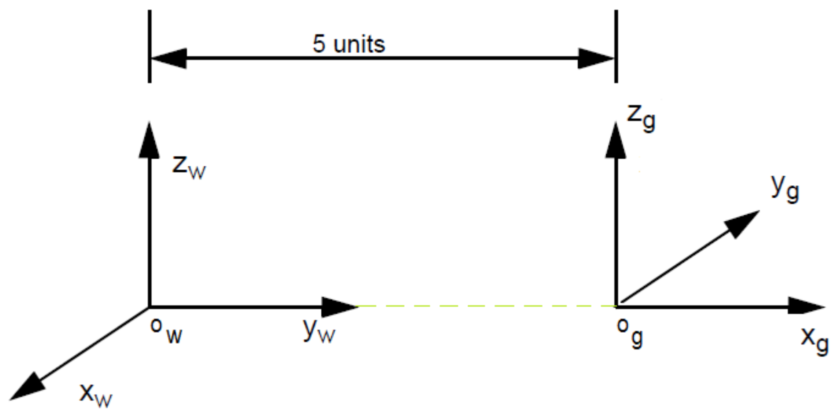
	1	2	3	4	5	6
1		■	■			
2			■	■	⊛	
3				■		
4			■			
5						■
6					■	
7	■	■				

2. For the same problem, initial position and goal execute the first two iterations of the A^* planner. Each move of the robot carries a cost of 1 units. For heuristic distance to goal, use the Manhattan distance. Setup all required data structures and show how they evolve in each iteration. Again, the grid is repeated to facilitate calculations.

	1	2	3	4	5	6
1		■	■			
2			■	■	⊛	
3				■		
4			■			
5						■
6					■	
7	■	■				

Problem 3 [6+6+6=18 Points]

1. Refer to the figure below, where a frame mounted on a robotic gripper $o_g x_g y_g z_g$ is moved with respect to the world frame $o_w x_w y_w z_g$. Write down an element of $SE(3)$ that describes this transformation.



2. Which of the following algorithms would need a complete map of the workspace before execution?

   (a) Bug1.

   (b) Tangent Bug

   (c) Wavefront planner

   (d) Potential function based planner

   (e) A^*

3. Consider a 6 DoF robot. It is observed that no pair of rows of the Jacobian matrix is independent for any configuration of this robot. What is the dimension of this robot's workspace?