EE-562. Robot Motion Planning
Problem Set # 5

Spring 2014

Due Date : March 31, 2014
Total Points : 100.

Problem 1

Consider a planar mobile robot with configuration space $\mathbb{R}^2 \times S^1$. A steerable dish-type antenna is mounted at coordinates $[a_x, a_y]$ relative to the robot's centroid. Derive the forward kinematics to predict the movement of the antenna dish. Give expressions for both positions and velocities in antenna coordinates. Derive the conditions under which instantaneous end point motion of the dish may become impossible.

Problem 2

Construct an explicit navigation function for a Euclidean 2-dimensional configuration space with the following description.

1. Boundary given by a circular fence of radius 5.
2. Circular obstacles of radius 0.5 at $(1, 1)$ and $(1, -1)$.
3. Goal configuration $(-1, 0)$.

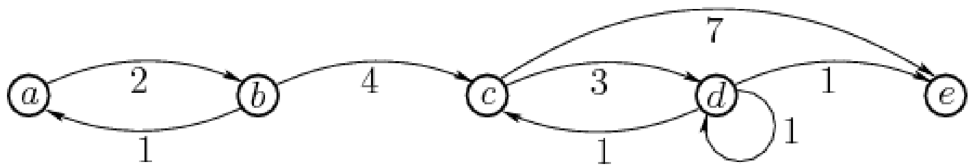
Try to sketch the contours of this navigation function for two different choices of tuning parameters.

Problem 3

Consider a 1D configuration space $\mathcal{Q} = [-1, 1]$ with boundaries at 1 and -1. Suppose your goal is $q = 0.7$. Compare the *naive* potential function

$$\phi_1(q) = (q - 0.7)^2,$$

against a properly constructed navigation function ϕ_2 using the method described in the class. Note that this is a 1D sphere world, with the world boundary defined by $\beta_0(q) = 1 - q^2$. Use both the analytical switch and a sharpening function to setup your function. For comparison, sketch the navigation functions ϕ_2 (for various values of tuning factors) against ϕ_1. Also, compute the gradient at the boundaries and comment on practical suitability of each option.



Problem 4

Consider a candidate potential function on S^1

$$\phi(\theta) = \frac{1 - \cos(\theta)}{3}$$

with a goal at $\theta = 0$. Assume no boundary on S^1. Would this be an appropriate navigation function as well? Compute the gradient and write an equation for the dynamics of the robot.

Problem 5

Consider the planning problem shown in Figure below. Let a be the initial state, and let e be the goal state.

1. Use backward value iteration to determine the stationary cost-to-go.
2. Do the same but instead use forward value iteration.

Refer to Section 2.3.1 of Lavalle's book for a similar example.