

Quiz 2

Marks: 10 + 3 Bonus

Wed, Nov 13, Fall 2013.

• **Problem 1**

The discrete-time state space pair (Φ, Γ) is said to be controllable if for any initial state \mathbf{x}_0 and any final state $\bar{\mathbf{x}}$, there exists a finite sequence of controls inputs u_0, u_1, \dots, u_{n-1} that transfers \mathbf{x}_0 to $\bar{\mathbf{x}}$.

1. Prove that the controllability of (Φ, Γ) is equivalent to the invertibility of the matrix

$$\mathcal{C} = [\Gamma \quad \Phi\Gamma \quad \Phi^2\Gamma \quad \dots \quad \Phi^{n-1}\Gamma].$$

Hint: The solution of a discrete-time state space difference equation is given by

$$\mathbf{x}_n = \Phi^n \mathbf{x}_0 + \sum_{m=0}^{n-1} \Phi^{n-m-1} \Gamma u_m. \quad (1)$$

2. Bonus: Prove that the invertibility of \mathcal{C} is equivalent to the invertibility of the matrix

$$\mathbf{W} = \sum_{m=0}^{n-1} \Phi^m \Gamma \Gamma^T (\Phi^T)^m. \quad (2)$$
