EE 561: Digital Control Systems

Problem Set 6

Due on Dec 6 at 5PM *Fall 2013*

- **Problem 1 [15 Points]**
  The state-matrices in Control Canonical form for a system are given by

$$\mathbf{\Phi}_c = \begin{bmatrix} 6 & -13 & 10 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}, \ \mathbf{\Gamma}_c = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \ \mathbf{C}_c = \begin{bmatrix} 1 & -4 & 3 \end{bmatrix}, \ \mathbf{D}_c = 0.$$

  Convert these matrices into Modal Canonical form having the new matrices $\mathbf{\Phi}_m, \mathbf{\Gamma}_m, \mathbf{C}_m$ and $\mathbf{D}_m$.

- **Problem 2 [20 Points]**
  Find out the state $\mathbf{x}^* = [x_1^* \ \ x_2^*]^\top$ and the input $\mathbf{u}^* = [u_1^* \ \ u_2^*]^\top$ that minimize the cost

$$J(x_1, x_2, u_1, u_2) = 2x_1^2 + 2x_1x_2 + 3x_2^2 + 5u_1^2 + 3u_1u_2 + 7u_2^2,$$

  to drive a system that is subject to the constraints

$$x_1 = 1 - x_2,$$
$$u_1 = 1 + u_2.$$

  The only stationary point of $J(x_1, x_2, u_1, u_2)$ is its one global minimum. Why there are no other stationary points such as local minima/maxima, global maxima or saddle points?

- **Problem 3 [30 Points]**
  For a plant with state $\mathbf{x}(t)$ and state matrices

$$\mathbf{F} = \begin{bmatrix} 0 & 1 \\ -3 & -5 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ \mathbf{J} = 0,$$

  (a) Design a controller, without using integral control, to yield a 5% overshoot and a settling time of 1 s.
  (b) Find out the steady-state error for a unit-step input, with this controller.
  (c) Now design the controller using integral control.
  (d) Find out the steady state error with this integral control, for a unit-step input. What 'type' of system is this?

- **Problem 4 [35 Points]**
  The transfer function for a system is given by

$$G(z) = \frac{z + 0.967}{z^2 - 1.67z + 0.91}.$$

  (a) Find control feedback $\mathbf{K}$ and observer gain $\mathbf{L}$ that place control poles at $z = 0.8 \pm j0.2$ and observer poles at $z = 0.6 \pm j0.3$.
  (b) Draw the block-diagram of the system in state-command structure.
  (c) For the state-command structure, find out $\mathbf{N}_x$ and $\mathbf{N}_u$, and plot $y(k)$ for a unit step input.
  (d) With $\bar{\mathbf{N}} = \mathbf{K}\mathbf{N}_x + \mathbf{N}_u$, draw the new block diagram substituting $\bar{\mathbf{N}}$ for $\mathbf{N}_x$ and $\mathbf{N}_u$.