EE 561: Digital Control Systems

Problem Set 5

Due on Nov. 25 in class

Fall 2013

- **Problem 1 [15 Points]**

  For this problem, refer to section 6.3.2 of 'Digital Control of Dynamic Systems' by Franklin et al.. A continuous-time system with state $\mathbf{x}$ is described by the state matrices

$$\mathbf{F} = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix}, \ \mathbf{J} = 0.$$

  (a) Find out the transfer function $H(s)$ for the system.

  (b) Discretize the system using a First-order Hold, and find out the new overall transfer function $G(z)$ for $T = 1$.

  (c) Now find out the new state matrices $\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{C_D}$ and $\mathbf{D_D}$. You can verify your result using `sysd = c2d(sys,T,'foh')` command on Matlab.

- **Problem 2 [15 Points]**

  A continuous-time system with state $\mathbf{x}$ is described by the state matrices

$$\mathbf{F} = \begin{bmatrix} -2 & 0 \\ -2 & 1 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ \mathbf{J} = 0,$$

  Discretize the system using Tustin's approximation, and find the new matrices $\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{C}$ and $\mathbf{D}$ for the new system

$$\begin{aligned} \mathbf{w}(k+1) &= \mathbf{\Phi}\mathbf{w}(k) + \mathbf{\Gamma}u(k) \\ y(k) &= \mathbf{C}\mathbf{w}(k) + \mathbf{D}u(k), \end{aligned}$$

  with the new state variable $\mathbf{w}(k) = \frac{1}{\sqrt{T}}\left(\mathbf{I} - \frac{T}{2}\mathbf{F}\right)\mathbf{x}(k) - \frac{\sqrt{T}}{2}\mathbf{G}u(k)$. You can verify your result using `sysd = c2d(sys,T,'tustin')` command on Matlab.

- **Problem 3 [20 Points]**

  The dynamic behavior of water-level in an irrigation channel shown in Fig. 1 is modeled by the differential equation

$$\dot{y}(t) = c_{\text{in}}u_{\text{in}}(t-\tau) - c_{\text{out}}u_{\text{out}}(t), \tag{1}$$

  where $y(t)$ is the downstream end water level variation; $u_{\text{in}}(t)$ measures inflow of water; $u_{\text{out}}(t)$ measures outflow of water; c_{in} and c_{out} are static gains; τ is the time taken by the water to cover the distance from A to B. Use the parameters $c_{\text{in}} = 10, c_{\text{out}} = 0$ (gate is closed), $\tau = 300$.

  (a) For value of the sampling interval $T = 60$, discretize the system using Forward Euler.

  (b) For this part, refer to section 4.3.4 of 'Digital Control of Dynamic Systems' by Franklin et al.. Convert the discretized system into state-space representation, by incorporating the delay τ, and find out the new matrices $\mathbf{\Phi}, \mathbf{\Gamma}, \mathbf{C_D}$ and $\mathbf{D_D}$.

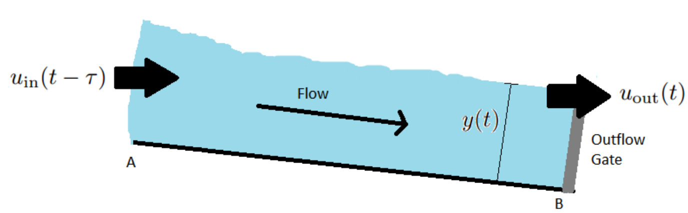

Figure 1: Irrigation channel model for Problem 3.

- **Problem 4 [35 Points]**
Consider the generalized transfer function

$$H(z) = \frac{Y(z)}{U(z)} = \frac{b_0 + b_1 z^{-1} + ... + b_n z^{-n}}{1 + a_1 z^{-1} + ... + a_n z^{-n}},$$

where $b_0 + b_1 z^{-1} + ... + b_n z^{-n} = b(z)$, and $1 + a_1 z^{-1} + ... + a_n z^{-n} = a(z)$.

(a) For the discrete-time system

$$\mathbf{x}(k+1) = \mathbf{\Phi_c}\mathbf{x}(k) + \mathbf{\Gamma_c}u(k)$$
$$y(k) = \mathbf{C_c}\mathbf{x}(k) + \mathbf{D_c}u(k),$$

derive the matrices $\mathbf{\Phi_c}, \mathbf{\Gamma_c}, \mathbf{C_c}$ and $\mathbf{D_c}$ for the Control Canonical form. You can use the states $X_i(z) = z^{-i}Q(z)$, where $Q(z) = \frac{U(z)}{a(z)} = \frac{Y(z)}{b(z)}$.

(b) Draw the block diagram for the system in Control Canonical form.

(c) Draw the dual of the block diagram in (b). [Hint: reverse the direction of all arrows, replace summers by junctions and junctions by summers. Note that the input and output need to be swapped.]

(d) Using new states (outputs of unit delays) derive the matrices $\mathbf{\Phi_o}, \mathbf{\Gamma_o}, \mathbf{C_o}$ and $\mathbf{D_o}$ for the Observer Canonical form.

(e) Now using the states $X_i(z) = \frac{1}{z-p_i}U(z)$, derive the matrices $\mathbf{\Phi_M}, \mathbf{\Gamma_M}, \mathbf{C_M}$ and $\mathbf{D_M}$ for Modal Canonical form in terms of c_i and p_i, where

$$c_i = \lim_{z \to p_i} \left[\frac{Y(z)}{U(z)} (z - p_i) \right]$$

and p_i are the roots of the equation $a(z) = 0$.

For this problem, you can seek guidance from section 4.2.3 of 'Digital Control of Dynamic Systems' by Franklin et al..

- **Problem 5 [15 Points]**
Transfer function of a plant is given by

$$H(s) = \frac{10}{(s+1)(s+2)}.$$

(a) Convert $H(s)$ into Observer Canonical form of discrete-time state space representation, using Matched Pole-Zero method (MPZ, recall from the mid-term and from the lecture on Nov. 11)

(b) Find out the polynomial $\alpha(s)$ to yield a 10% overshoot with a settling time of 0.5 s. Discretize $\alpha(s)$ through MPZ method, and design a discrete-time predictive observer for the system.