EE 561: Digital Control Systems

Problem Set 4

Due on Mon. 4^{th} Nov. in class *Fall 2013*

- **Problem 1 [30 Points]**

  A certain system with state $\mathbf{x}$ is described by the state matrices

$$\mathbf{F} = \begin{bmatrix} -2 & 0 \\ -2 & 1 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 3 \\ 1 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \ \mathbf{J} = 0,$$

  (a) Compute the eigenvalues and eigenvectors of the matrix $\mathbf{F}$. Write down a diagonal matrix $\mathbf{\Lambda}$ and a matrix $\mathbf{V}$ such that $\mathbf{F} = \mathbf{V\Lambda V^{-1}}$. (Hint: recall 'eigendecomposition' from your Linear Algebra course.)

  (b) Using the results from (a), find out the transformation $\mathbf{T}$ so that if $\mathbf{x} = \mathbf{Tz}$, the state matrices describing the dynamics of $\mathbf{z}$ are in Modal Canonical form.

  (c) Compute the new matrices $\mathbf{A}, \mathbf{B}, \mathbf{C},$ and $\mathbf{D}$.

  (d) Compare the matrices $\mathbf{A}$ and $\mathbf{\Lambda}$.

  (e) Plot the step-response of the system using the following commands in Matlab

```
>>sys = ss(A, B, C, D)
>>step(sys)
```

  (f) Write down the total solution $\mathbf{x}(t)$ of the system.

  (g) Discretize the total solution with sampling period T to convert the state-equations into the following form

$$\mathbf{x}_{k+1} = \mathbf{\Phi}\mathbf{x}_k + \mathbf{\Gamma}u_k,$$
$$y_k = \mathbf{H}\mathbf{x}_k,$$

  where $\mathbf{\Phi}$ and $\mathbf{\Gamma}$ need to be computed.

  (h) Find out the discretized transfer function $H(z)$ of the system.

  (i) Plot the step-response of the discretized system with $T = 0.01$ s. You can use the Matlab function `c2d(sys,T)` to discretize the system.

  (j) Compare the step-response obtained in (i) with the one obtained in (e). Comment on any similarities and/or differences.

- **Problem 2 [20 Points]**

  Transfer function of a plant is given by

$$G(s) = \frac{10}{(s+1)(s+2)}.$$

  (a) Using 'comparing the coefficients' method, design a state feedback (controller) for this plant to yield a 15% overshoot with a settling time of 0.5 s.

  (b) Repeat (a) using Ackermann's control formula for pole placement.

  (c) Plot the step-response of the system and verify that the required specifications have been successfully met.

- **Problem 3 [10 Points]**

  Determine whether the system with state $\mathbf{x}$ and described by the state matrices

$$\mathbf{F} = \begin{bmatrix} -2 & -1 & -3 \\ 0 & -2 & 1 \\ -7 & -8 & -9 \end{bmatrix}, \ \mathbf{G} = \begin{bmatrix} 2 \\ 1 \\ 2 \end{bmatrix}, \ \mathbf{H} = \begin{bmatrix} 4 & 6 & 8 \end{bmatrix}, \ \mathbf{J} = 0,$$

  is observable. Verify your answer using the following commands in Matlab

```
>>ObsMat = obsv(F, H)
>>Rank = rank(ObsMat)
```

- **Problem 4 [10 Points]**
  The plant's transfer function of a model for the body's blood glucose level is given below,

$$G(s) = \frac{407(s + 0.916)}{(s + 1.27)(s + 2.69)}.$$

  Design an observer for this plant to meet the specifications $\zeta = 0.7$ and $\omega_n = 100$ for the closed-loop system.

- **Problem 5 [30 Points]**
  The plant for the antenna azimuth position control system is given below,

$$G(s) = \frac{1325}{s(s + 1.71)(s + 100)}.$$

  A controller needs to be designed to meet certain design specifications. And because the state variables of the plant are not accessible, an observer also needs to be designed to estimate the states.

  (a) Write down the state matrices $\mathbf{A}, \mathbf{B}, \mathbf{C},$ and $\mathbf{D}$ for the plant.

  (b) Draw a block diagram of the complete control system including plant, controller and observer.

  (c) Determine the controllability of the system. Show your working clearly.

  (d) If the system is controllable, design a controller to yield a 10% overshoot and a settling time of 1 s. Place the third pole 10 times as far from the imaginary axis as the second-order dominant pole.

  (e) Determine the observability of the system. Show your working clearly.

  (f) If the system is observable, design an observer whose transient response has 10% overshoot and a natural frequency 10 times as great as the natural frequency of the response in (d). Place the third pole 10 times as far from the imaginary axis as the observer's second-order dominant pole.

  (g) Using Matlab, plot the step-response of the system using zero initial conditions. Also plot the estimated output $\hat{y}$ on the same graph.

  (h) Now plot the step-response of the system assuming the initial condition $x_1 = 0.006$ at $t = 0$. Also plot the estimated output $\hat{y}$ on the same graph using zero initial conditions.

  (i) How long does it take for the output error to decay below 1%?