EE 561: Digital Control Systems

Problem Set 3

Due on Mon. 7[th] Oct. in class. *Fall 2013*

- **Problem 1**

For the causal LTI system described by the difference equation

$$y_k + \frac{1}{2}\, y_{k-1} = x_k, \tag{1}$$

(a) By first finding out the z-transform and then applying a suitable substitution for z, determine the frequency response $H(e^{j\omega})$ for the system. Also find out the frequency response against unnormalized frequency if the sampling frequency $f_s = 10$ Hz.

(b) Find out y_k, for $x_k = \delta[k]$, and determine its discrete-time Fourier transform (DTFT) using the formula

$$Y(e^{j\omega}) = \sum_{k=-\infty}^{+\infty} y_k e^{-j\omega k}. \tag{2}$$

(c) Compare the results of (a) and (b), and state the reason for any similarities or differences between $H(e^{j\omega})$ and $Y(e^{j\omega})$.

- **Problem 2**

(a) For the Euler Forward s-to-z approximation $z \simeq 1 + Ts$, find out the range of values of s (if any) in terms of T for which

(i) a stable pole on the real axis in s-domain becomes unstable in z-domain.

(ii) an unstable pole on the real axis in s-domain becomes stable in z-domain.

(b) Repeat (a) for the Euler Backward s-to-z approximation $z \simeq \dfrac{1}{1 - Ts}$.

(c) For $H(s) = \dfrac{s+1}{s^2 - 9}$, transform poles and zeros to z-domain using each of Euler Forward, Euler Backward and Tustin's $\left(z \simeq \dfrac{2 + Ts}{2 - Ts}\right)$ methods. Comment on the change in stability of the system for each case, while varying T from 0 to 1; for example, using $T = 0.01,\ 0.2,\ 0.5,\ 0.8,$ and 1. Also suggest which approximation method best preserves the system's stability characteristics.

- **Problem 3**

In the schematic shown in Fig. 1, assume that the mass of the spacecraft plus gas tank, m_1, is 2000 kg and the mass of the probe, m_2, is 1000 kg. A rotor will float inside the probe and will be forced to follow the probe with a capacitive forcing mechanism. The spring constant of the coupling k is 4.2×10^6. The viscous damping b is 5.6×10^3.

(a) Write the equation of motion for the system consisting of masses m_1 and m_2 using the inertial position variables, y_1 and y_2.

(b) The actual disturbance u is a micrometeorite, and the resulting motion is very small. Therefore, rewrite your variables with scaled variables $z_1 = 10^6 y_1$, $z_2 = 10^6 y_2$, and $v = 1000u$.

(c) Put the equations in state-variable form using the state $\mathbf{x} = [z_1 \;\; \dot{z}_1 \;\; z_2 \;\; \dot{z}_2]^T$, the output $y = z_2$, and the input an impulse $u = 10^{-3}\delta(t)$ Ns on mass m_1.

(d) Using the numerical values, enter the equations of motion into MATLAB in the form

$$\dot{\mathbf{x}} = \mathbf{F}\mathbf{x} + \mathbf{G}v, \tag{3}$$

$$y = \mathbf{H}\mathbf{x} + Jv, \tag{4}$$

and define the MATLAB system: `sysGPB = ss(F,G,H,J)`. Plot the response of y caused by the impulse with the MATLAB command `impulse(sysGPB)`. This is the signal the rotor must follow.

(e) Use the MATLAB command `p = eig(F)` to find out the poles (or roots) of the system and roots `z = tzero(F,G,H,J)` to find out the zeros of the system.

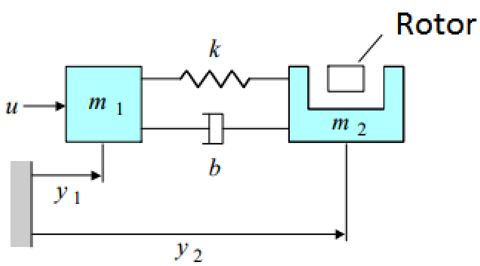

Figure 1: Mechanical set-up for Problem 3.

- **Problem 4**

Give the state description matrices in control-canonical form for the following transfer functions:

(a) $\dfrac{1}{3s+2}$

(b) $\dfrac{4s+1}{s^2+5s+4}$

(c) $\dfrac{(s+8)(s^2+s+25)}{s^2(s+5)(s^2+s+36)}$

- **Problem 5**

For the system with state $\mathbf{x}$ and described by the state matrices

$$\mathbf{F} = \begin{bmatrix} -2 & 1 \\ -2 & 0 \end{bmatrix}, \; \mathbf{G} = \begin{bmatrix} 1 \\ 3 \end{bmatrix}, \; \mathbf{H} = \begin{bmatrix} 1 & 0 \end{bmatrix}, \; J = 0,$$

find out the transformation $\mathbf{T}$ so that if $\mathbf{x} = \mathbf{Tz}$, the state matrices describing the dynamics of $\mathbf{z}$ are in control canonical form. Compute the new matrices $\mathbf{A}, \mathbf{B}, \mathbf{C},$ and $\mathbf{D}$.

- **Problem 6**

For the system with state $\mathbf{x}$ and described by the state matrices

$$\mathbf{F} = \begin{bmatrix} -4 & 1 \\ -2 & -1 \end{bmatrix}, \; \mathbf{G} = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

find out the steady-state value of the step-response assuming zero initial conditions.

- **Problem 7**

For the system shown in Fig. 2:

(a) Find out the transfer function from U to Y.

(b) Write state equations for the system using the state variables indicated.

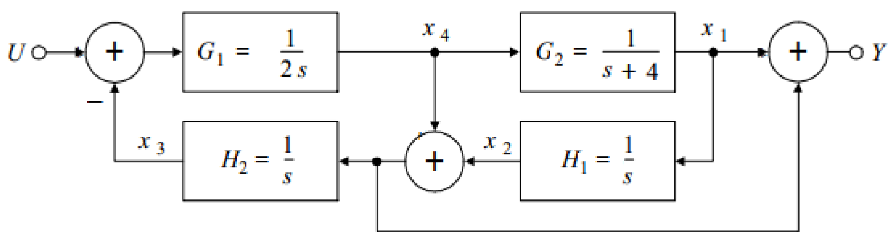

Figure 2: Block diagram for Problem 7.

- **Problem 8**

  Consider the circuit in Fig. 3, with an input voltage source $u(t)$ and an output current $y(t)$.

  (a) Using capacitor voltage and inductor current as state variables, write state and output equations of the system.

  (b) Find out the conditions relating R_1, R_2, C, and L that render the system uncontrollable.

  (c) Physically interpret the conditions found in (b) in terms of the time constants of the system.

  (d) Find out the transfer function of the system.

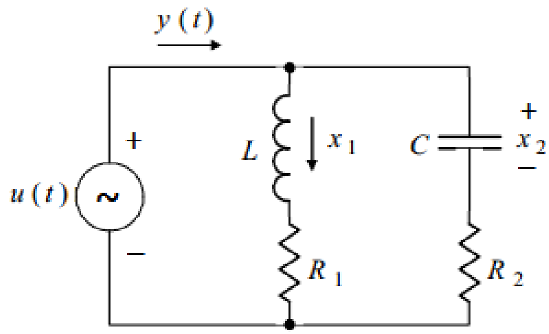

Figure 3: Circuit diagram for Problem 8.