EE 561: Digital Control Systems

Problem Set 2

Due on Wed. 25th Sept. *Fall 2013*

- **Problem 1**
  Check the following for (internal) stability. [Hint: Analyze the characteristic equation.]
  (a) $u_k = 0.5u_{k-1} - 0.3u_{k-2}$
  (b) $u_k = 1.6u_{k-1} - u_{k-2}$
  (c) $u_k = 0.8u_{k-1} + 0.4u_{k-2}$

- **Problem 2**
  The first-order system $\frac{z-\alpha}{(1-\alpha)z}$ has a zero at $z = \alpha$.
  (a) Plot the step response for this system for $\alpha = 0.8, 0.9, 1.1, 1.2, 2$.
  (b) Plot the overshoot of the system on the same coordinates as those appearing in Fig. 1 for $-1 < \alpha < 1$.
  (c) In what way is the step response of this system unusual for $\alpha > 1$?

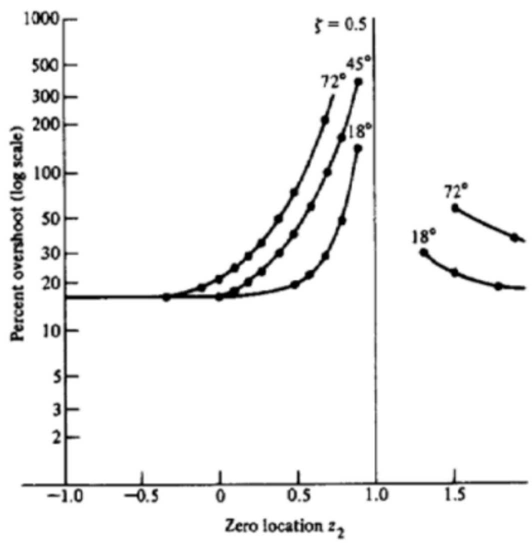

Figure 1:

- **Problem 3**
  Show that the characteristic equation

$$z^2 - 2r\cos(\theta)z + r^2$$

  has the roots

$$z_{1,2} = re^{\pm j\theta}.$$

- **Problem 4**
  For a second-order system with damping ratio 0.5 and poles at an angle in the z-plane of $\theta = 30°$, what percent overshoot to a step would you expect if the system had a zero at $z_2 = 0.6$? [Hint: You can either analyze it geometrically, or you can also estimate it from the graph in Fig. 1.]

- **Problem 5**

  Sketch the inverse z-transform, f_k, for each of the following transforms, for up to at least three time constants. You can also use an inverse z-transform table.

  (a) $F(z) = \dfrac{1}{1+z^{-2}}, \qquad |z| > 1;$

  (b) $F(z) = \dfrac{z(z-1)}{z^2 - 1.25z + 0.25}, \qquad |z| > 1;$

  (c) $F(z) = \dfrac{z}{z^2 - 2z + 1}, \qquad |z| > 1;$

  (d) $F(z) = \dfrac{z}{(z-\frac{1}{2})(z-2)}, \qquad |z| > 2.$

- **Problem 6**

  Use the z-transform to solve the difference equation

$$y_k - 3y_{k-1} + 2y_{k-2} = 2u_{k-1} - 2u_{k-2}.$$

$$u_k = \begin{cases} k, & k \geq 0, \\ 0, & k < 0, \end{cases}$$

$$y_k = 0, \quad k < 0.$$

- **Problem 7**

  For this problem, refer to section 6.6.1 of Signals and Systems (2e) by Oppenheim et. al.. For the first-order causal LTI system described by the difference equation (warning: this is not the same equation that appears in section 6.6.1)

$$y_k + ay_{k-1} = ax_k.$$

  (a) Find out the z-transform $H(z)$ of the system.

  (b) Using a suitable substitution to the z-transform found in (a), find out the frequency response $H(e^{j\omega})$ of the system.

  (c) For $a = -3/2, -1/3, 0, 1/3,$ and $3/2$,

    (i) Plot the step responses of the system.

    (ii) Plot the magnitude responses of the system. Comment on the variation in shape of the graph with the variation in a.

- **Problem 8**

  For this problem, refer to section 6.6.2 of Signals and Systems (2e) by Oppenheim et. al.. For the second-order causal LTI system described by

$$y_k - 2r\cos(\theta)y_{k-1} + r^2 y_{k-2} = x_k,$$

  with $0 \leq \theta \leq \pi$.

  (a) Find out the z-transform $H(z)$ of the system.

  (b) Find out the frequency response $H(e^{j\omega})$ of the system.

  (c) For $(r,\theta) = (2/3, 0), (3/2, \pi/4), (1/3, \pi/2), (2/3, 3\pi/4), (2/3, \pi)$,

    (i) Plot the step responses of the system. Also classify each response as either undamped, underdamped, critically damped or overdamped.

    (ii) Plot the magnitude responses of the system. Comment on the variation in shape of the graph with the variation in r and θ.