

# EE 561: Digital Control Systems

Spring 2017

## Quiz # 1

**Time: 30 min**

**Name:** \_\_\_\_\_

**Roll #:** \_\_\_\_\_

### Question # 1

**(10 marks)**

In class we derived, the discrete-time Z.O.H equivalent for a continuous-time system  $G(s)$  as follows

$$G(z) = (1 - z^{-1})\mathcal{Z}\left\{\frac{G(s)}{s}\right\} \quad (1)$$

Now consider a first-order hold operation where the output  $u(t)$  for time  $(k-1)T < t \leq kT$  is given by the straight line obtained by extrapolating the values of the discrete-time input  $u_k$  at  $k-1$  and  $k-2$ . Thus the output of the first order hold in graphical form, when the input is a discrete impulse  $\delta_k$  is given by the figure shown below.

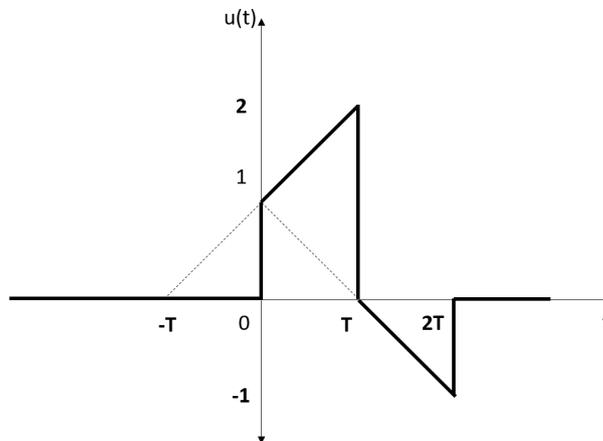


Figure 1: Response of the first order hold to a discrete impulse.

Derive an expression similar to (1) for the first order hold equivalent of  $G(s)$ .

**Hint:** begin by expressing the output  $u(t)$  as a linear combination of the functions  $\mathbb{1}(t)$ ,  $\mathbb{1}(t - T)$  and  $\mathbb{1}(t - 2T)$ .

**Question # 2****(10 marks)**

Consider the following form of the P.I.D control implemented in continuous time

$$u(t) = K \left[ e(t) + \frac{1}{T_I} \int_0^t e(\eta) d\eta + T_D \dot{e}(t) \right] \quad (2)$$

the z-transform of the corresponding control in discrete-time may be given by the following expression

$$U(z) = \left[ K_P + \frac{K_I}{1 - z^{-1}} + K_D(1 - z^{-1}) \right] E(z) \quad (3)$$

Your task is to obtain (2) as an approximation of (3) when the integral term is approximated by the trapezoidal rule and the derivative is approximated by the backward difference. What are the values of  $K_P$ ,  $K_I$  and  $K_D$  in terms of the original weights  $K$ ,  $T_I$  and  $T_D$ ? Assume that the sampling period is given by  $T$ .

**Question # 3****(10 marks)**

Consider an instance of the P.I.D controller given below

$$U(z) = \left[ 1 + \frac{2}{1 - z^{-1}} + 3(1 - z^{-1}) \right] E(z)$$

write down a pseudo-code that implements the controller by using no more than 2 memory elements.