EE 561: Digital Control Systems
Problem Set # 4
Spring 2017

Due date: May 04, 2017 **Total marks: 70**

Question # 1 (25 marks)

Consider the following continuous-time system

$$G(s) = \frac{Y(s)}{U(s)} = \frac{1}{s(s+2)}$$

a) (5-marks)

Obtain a state space representation of the given system by selecting $x_1 = y$ and $x_2 = \dot{x}_1$.

b) (10-marks)

Calculate the discrete-time equivalent of the state space representation obtained above. Assume that the control is applied through a zero-order-hold, and that the sampling time $T = 1$.

c) (5-marks)

Determine the discrete time transfer function $G(z) = Y(z)/U(z)$.

d) (5-marks)

Show that $G(z)$ obtained above is the same as the one obtained through the Z.O.H equivalent of $G(s)$ i.e., $G(z) = \mathcal{Z}\left[(1 - e^{-Ts})\frac{G(s)}{s}\right]$.

Question # 2 (25 marks)

Consider the following system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 1 & T \\ 0 & 10 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} T^2/2 \\ T \end{bmatrix} u(k); \quad y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

a) (10-marks)

Design a full-state prediction observer of the system where it is desired that all observer poles be zero.

b) (10 marks)

Write down the dynamics for the estimator error. Then assume that the initial state $x(0) = [a_1\ b_1]^T$ and the initial estimate $\bar{x}(0) = [a_2\ b_2]^T$. Show that the estimation error goes to zero in at most two sampling periods. (**hint:** solve the dynamics of the error recursively, to compute $\tilde{x}(2)$).

c) (5 marks)

A response such as the one obtained above that attains a value of exactly zero in finite time is called a dead-beat response. The concept of a dead-beat response is unique to discrete-time systems and has no counterpart in continuous time. This can only happen if the response dynamics are given by a nilpotent matrix.
Show that a matrix with every eigenvalue zero (such as the one designed above), is nilpotent.

Question # 3 (20 marks)

Consider the system

$$\begin{bmatrix} x_1(k+1) \\ x_2(k+1) \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -0.16 & -1 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u(k); \quad y(k) = \begin{bmatrix} 1 & 0 \end{bmatrix} \begin{bmatrix} x_1(k) \\ x_2(k) \end{bmatrix}$$

Assume that the system is designed to track a reference input and the control scheme is given as $u(k) = Nr(k) - Kx(k)$, where N and K are design parameters (assume $r(k)$ is a scalar).

a) (10-marks)

Determine the gain matrix K, if the desired characteristic polynomial of the closed-loop poles is $\alpha_c(z) = z^2 - z + 0.5$. You may assume that $r(k) = 0$ in this step.

b) (5 marks)

Now, after incorporating the control law $u(k) = Nr(k) - Kx(k)$ with the gains K chosen as above, determine the transfer function $Y(z)/R(z)$.

c) (5 marks)

Using the result of part b, and the final value theorem in discrete-time, choose the design parameter N so that the steady-state value of the step-response is unity (the steady state error is zero).