EE 561: Digital Control Systems
Problem Set # 3
Spring 2017

Due date: Apr 20, 2017 **Total marks: 180**

Question # 1 (40 marks)

Obtain diagonal representations of the following systems. For systems that are diagonalizable, transform the system to modal canonical form. For diagonal systems with complex entries, obtain an equivalent block-diagonal, real-valued representation. For systems that are not diagonalizable, find the corresponding Jordan form.

(**Note:** You are encouraged to check your results in MATLAB. However, you must give the complete calculations here in order to obtain full credit.)

a) (10-marks)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 2 & 3 \\ 1 & 3 & 3 \\ -1 & -2 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} u; \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

b) (10-marks)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 2 & 4 & -8 \\ 0 & 0 & 4 \\ 0 & -1 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} u; \quad y = \begin{bmatrix} 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

c) (10-marks)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 4 & 0 & 1 \\ 2 & 3 & 2 \\ 1 & 0 & 4 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u; \quad y = \begin{bmatrix} 1 & 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

d) (10-marks)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 1 \\ 2 & 1 & -1 \\ -6 & -5 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} u; \quad y = \begin{bmatrix} 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

e) (10-marks)

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \\ \dot{x}_4 \end{bmatrix} = \begin{bmatrix} 2 & 9 & 0 & 2 \\ -1 & 2 & 1 & 0 \\ 0 & 0 & 3 & 0 \\ 0 & 0 & 1 & -1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix} u; \quad y = \begin{bmatrix} 1 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{bmatrix}$$

Question # 2 (40 marks)

Before attempting this question, consider the following definitions

Definition 1: A system is *stabilizable* if the unstable modes are controllable or, equivalently, the uncontrollable modes are stable.

Definition 2: A system is *detectable* if the unstable modes are observable or, equivalently, the unobservable modes are stable.

Given the above definitions, for each system given in Question 1 do the following

a) (20-marks)
Determine if the system is controllable. If not, specify the uncontrollable modes. Is the system stabilizable?

b) (20-marks)
Determine if the system is observable. If not, specify the unobservable modes. Is the system detectable?

Question # 3 (20 marks)

Consider a plant with transfer function

$$\frac{Y(s)}{U(s)} = \frac{10}{(s+1)(s+2)(s+3)}$$

a) (5-marks)
Obtain the state space representation of the system by defining state variables as $x_1 = y$,

$x_2 = \dot{x}_1$ and $x_3 = \dot{x}_2$.

b) (15-marks)
Using pole-placement via full state-feedback, i.e., $u = -Kx$, place the closed loop poles at

$$p_{1,2} = -2 \pm j2\sqrt{3}, \quad p_3 = -10$$

What is the required feedback gain matrix K?

Question # 4 (25 marks)

Consider the following system

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 20.6 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 \\ 1 \end{bmatrix} u; \quad y = \begin{bmatrix} 0 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

a) (10-marks)
Design a full-order state observer so that the eigenvalues of the observer matrix are given by $\lambda_1 = \lambda_2 = -10$. What are the observer gains?

b) (15-marks)
For the initial condition $x_1(0) = 0.1$, $x_2(0) = 2$ and a unit step control $u(t) = \mathbb{1}(t)$, use a plot of the error to determine whether or not the estimate converges. Attach your MATLAB code along with the plot.

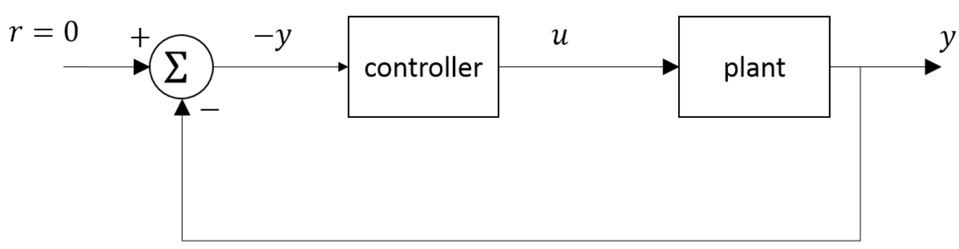

Figure 1: Regulator system for Question 5.

Question # 5 **(55 marks)**

Consider the regulator system shown in Figure 1. The plant transfer function is

$$G(s) = \frac{Y(s)}{U(s)} = \frac{10(s+2)}{s(s+4)(s+6)}$$

a) (5-marks)

Derive a state space representation of the plant by selecting the following states

$$x_1 = y; \quad x_2 = \dot{x}_1; \quad x_3 = \dot{x}_2 - 10u$$

After presenting the state-space matrices, check their validity by showing that the corresponding transfer function is indeed equal to $G(s)$.

b) (20-marks)

Design a compensator so that the closed loop poles are placed at

$$p^c_{1,2} = -1 \pm j2, \quad p^c_3 = -5$$

and the poles of the reduced-order observer (we need not estimate x_1) are located at

$$p^o_1 = -4.5, \quad p^o_2 = -4.5$$

c) (10-marks)

Show that the transfer function of the compensator is given by

$$G_c(s) = \frac{1.2109(s+5.36)(s+3.9)}{(s+5.62)(s+0.38)}.$$

To see how to calculate the transfer function for combined control-law and reduced-order estimation, refer to Equation (7.181) in Section 7.8 of "Feedback Control of Dynamic Systems" by Franklin et. al.

d) (20-marks)

Obtain the response of the compensated system with the following initial conditions

$$x(0) = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \quad \tilde{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

where $x(0)$ and $\tilde{x}(0)$ are initial conditions for the state and observer error respectively. Attach plots for $x_1(t)$, $x_2(t)$, $x_3(t)$, $\tilde{x}_1(t)$ and $\tilde{x}_2(t)$. Clearly show any calculations and attach the MATLAB code used to obtain the plots.