EE 561: Digital Control Systems
Problem Set # 2
Spring 2017

Due date: Mar 07, 2017

Question # 1 **(20 marks)**

Investigate the internal and BIBO stability of the systems described respectively by the following recurrences

a) $u_{k+3} + 2u_{k+2} + 1.5u_{k+1} + 0.5u_k = e_{k+1}$

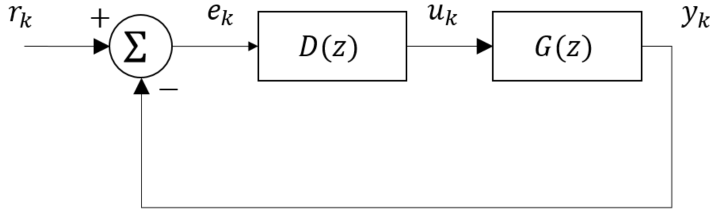
b) $u_k - u_{k-1} + 0.21u_{k-2} = 2e_{k-1} - 3e_{k-2}$

c) $u_{k+2} + 2.5u_{k+1} + u_k = e_{k+1} + 2e_k$

d) $u_{k+2} - u_{k+1} + u_k = 3e_{k+1} + 3e_k$

Question # 2 **(10 marks)**

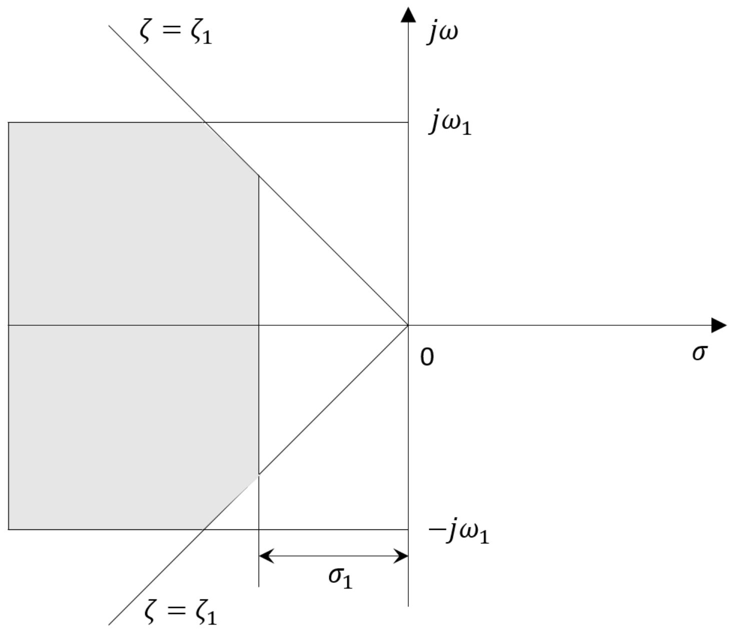
Consider the following system.



Find an expression in terms of $G(z)$ and $D(z)$ for the *error transmittance* $T_E(z)$ such that $E(z) = T_E(z)R(z)$.

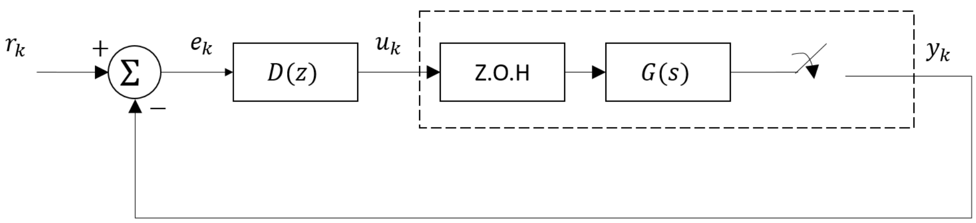
Question # 3 **(10 marks)**

Using the map $z = e^{sT}$, plot the region in the z-plane that corresponds to the desired region in the s-plane given by the shaded area in the following figure.



Question # 4 **(25 marks)**

Consider the following sampled-data system with plant $G(s)$ preceded by a zero order hold, and a control law implemented by the controller $D(z)$.



a) (6-marks)
Obtain the Z.O.H equivalent $G(z)$ for $G(s)$ when

$$G(s) = \frac{1}{s+1}$$

b) (4-marks)
Using the hold equivalent obtained in part a), derive the discrete transfer function relating the output y_k to the input r_k, if the control law is defined by the following transfer function

$$D(z) = \frac{K}{1 - z^{-1}}$$

c) (7-marks)
For $K = 2$, and $T = 0.5$, plot the response y_k when the input r_k is a unit step. Repeat for $T = 1$ and $T = 2$ seconds. What effects do you observe as the sampling time is increased?

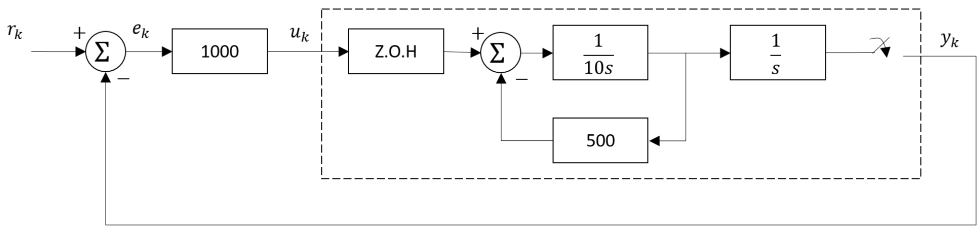
d) (8-marks)
For $K = 2$, and $T = 0.5$, use the final value theorem to calculate the steady state error when the input is a unit ramp. Repeat for $T = 1$ and $T = 2$ seconds (you may use the result obtained in Question 2). Explain the observed effect of sample-time increase.

Question # 5 **(20 marks)**

The long-term behavior of any system is so important that systems are commonly classified based on the steady state characteristics of their response to some basic signals. The classification yields what is called the "system type". Consider the following definitions

1. A system is said to be of Type-0, if it exhibits a finite steady state error in response to a step input and infinite steady state error to inputs of higher order.
2. A system is said to be of Type-1, if it exhibits no steady state error in response to step inputs, a finite steady state error to ramp inputs and infinite steady state error to inputs of higher order.
3. A system is said to be of Type-2, if it exhibits no steady state error in response to step and ramp inputs, a finite steady state error to parabolic inputs and infinite steady state error to inputs of higher order.

Consider the system given in the following figure



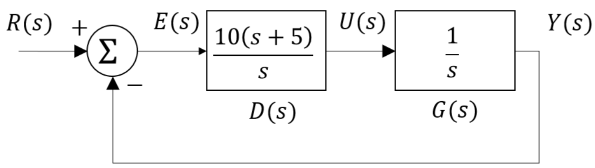
a) (6-marks)
Find the Z.O.H equivalent, relating $Y(z)$ to $U(z)$.

b) (4-marks)
Find the expression for the error transmittance relating $E(z)$ to $R(z)$. You may use the result obtained in Question 2

c) (10-marks)
If $T = 1$ seconds, use the final-value theorem to calculate the steady state error of the system to unit step, unit ramp and unit parabolic inputs. What is the system type?

Question # 6 **(15 marks)**

Consider the following system in continuous time where the controller $D(s)$ is designed to control the plat $G(s)$.



Here your task is to discretize the controller $D(s)$ by first using Tustin's approximation and then using the forward rectangular substitution rule. Assume that the controller operates at $20Hz$.
a) (6-marks)
Obtain expressions $D_T(z)$ and $D_{FR}(z)$ for the discretized controllers.

b) (9-marks) Now evaluate the performance of the approximations. To do this, plot the **continuous** step response of the system when

1. The plant is controlled by $D(s)$
2. The plant is controlled by $D_{FR}(z)$ with sample and hold (zero order).
3. The plant is controlled by $D_T(z)$ with sample and hold (zero order).

To plot the respective responses, you may implement the block-diagram in Simulink while adding the appropriate ADC ad DAC blocks. Which of the two discrete controllers gives a better approximation to the continuous-time response?