On the Structural Complexity of Multi-Agent Robot Formations

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***Abstract*— We present a complexity measure for studying the structural complexity of multi-agent robot formations. We base this measure on the total information flow in the system, which is due to sensory perception and communication among agents. We show that from an information theoretic point of view, perception and communication are fundamentally the same. We show how the information flows depends on different protocols and that the broadcast protocol corresponds to the worst case complexity for a given formation.**

I. Introduction

When designing control strategies for distributed, multi-agent robot systems, it is vitally important that the number of prescribed local interactions is managed in a scalable manner. In other words, it should be possible to add new robots to the system without causing a significant increase in the communication and computational burdens of the individual robots. On the other hand, an additional requirement when designing multi-agent coordination strategies should be that enough local interactions are present in order to ensure the proper execution of the task at hand.

Hence, a fundamental question that arises when studying such multi-agent systems is how to properly define the notion of "complexity". The traditional, algorithmic notion of the complexity of a system is related to how difficult it is to *describe* it. Therefore, most of the measures of complexity are closely related to the *Algorithmic Information Content* (AIC) in a system [1]. However, as noted in the molecular chemistry literature [2], [3], [4], there is an inherent difference between *descriptive complexity* and *structural complexity*, where the latter measures the interactions, size, and asymmetry in the physical structure. A similar program can be carried out within the context of formation control. It is clear that when talking about robot formations, any measure of the complexity of the formations should take into account the size of the formation, the number of communication links or interactions in the formation, and possibly also the degree of symmetry in the formation.

Molecular chemists have mainly described the structural complexity of molecules by defining measures on their corresponding graphs [2]. Fortunately, there is a corresponding notion of formation graphs, induced by robot formations, [5], [6], [7], [8], [9], [10], where the structural information in the formation is captured. Therefore, it seems appropriate to study the structural complexity of multi-agent robot formations with reference to their graphs. We will frequently refer to our work on *connectivity graphs* [5], [6], [7] of robot formations in order to make this notion concrete.

When formulating a measure of complexity for robot formations, it need not produce an absolute order on all connectivity graphs (although the order has to be observed in its own class e.g. among all rings, all stars, all complete graphs). This means that we are more interested in relative complexity. For this program to be considered successful, we should thus at least be able to differentiate between *very complex* formations and *very simple* ones.

Given the above mentioned considerations, we will define a complexity measure of robot formations, related to the complexity of its connectivity graphs. This paper is organized as follows: We will first introduce connectivity graphs of formations in Section II. Following this, we will discuss the equivalence between perception and communication from an information theoretic point of view, in Section III. Then, we will propose a definition of the intrinsic complexity of robot formations, in Section IV and explain its relation to the complexity of graphs.

II. Formations and Connectivity Graphs

In order to see how a graph-based complexity measure is appropriate when studying multi-agent formation, we, in this section, recall some previous results and definitions of connectivity graphs. The technical details can be found in [5], [6], [7] but we include this treatment for the sake of clarity. Throughout this paper it will be assumed that the robots are planar, and that they can interact with neighboring robots (through perception or communication) that are no further than δ away.

The configuration space $\mathcal{C}^N(\mathbb{R}^2)$ of the robot formation is made up of all ordered N-tuples in $\mathbb{R}^2$, with the property that no two points coincide. The evolution of the formation can be represented as a trajectory $\mathcal{F} : \mathbb{R}_+ \to \mathcal{C}^N(\mathbb{R}^2)$, usually written as $\mathcal{F}(t) = (X_1(t), X_2(t), \ldots X_N(t))$ to signify time evolution. The spatial relationship between robots can be represented as a graph in which the vertices of the graph represent the robots, and the pair of vertices on each edge tells us that the corresponding robots are within sensor range δ of each other.[1]

Definition 2.1 (Connectivity Graph of a Formation): Let $\mathcal{G}_N$ denote the space of all possible graphs that can

[1] Here, δ is used to signify the limited effective range of the sensors as well as the range within which a communication channel is available.

be formed on N vertices $V = \{v_1, v_2, \ldots, v_N\}$. Then we can define a function $\Phi_N : C^N(\mathbb{R}^2) \rightarrow \mathcal{G}_N$, with $\Phi_N(\mathcal{F}(t)) = \mathcal{G}(t)$, where $\mathcal{G}(t) = (\mathcal{V}, \mathcal{E}(t)) \in \mathcal{G}_N$ is the *connectivity graph* of the formation $\mathcal{F}(t)$. Furthermore, $v_i \in \mathcal{V}$ represents robot i at position X_i, and $\mathcal{E}(t)$ denotes the edges of the graph, with $e_{ij}(t) = e_{ji}(t) \in \mathcal{E}(t)$ if and only if $\|X_i(t) - X_j(t)\| \leq \delta$, $i \neq j$.

The movements of the individual robots in the formation may result in the removal or addition of edges in the graph. Therefore, $\mathcal{G}(t)$ is a dynamic structure. It is clear that different formations can produce a wide variety of graphs with N vertices. This includes graphs that have disconnected subgraphs, or totally disconnected graphs with no edges. However, the problem of switching between different formations or of finding interesting structures within a formations can only be tackled if no "sub-formations" of robots are completely isolated from the rest of the formation. This means that the connectivity graph $\mathcal{G}(t)$ of the formation $\mathcal{F}(t)$ should always remain *connected* (in the sense of connected graphs) for all time.

III. Perception Vs. Communication

Any measure of how complex a certain formation is has to capture the amount of information that flows between the different agents in a meaningful manner. This exchange of information between agents is due to the two types of local interactions among agents, one due to sensory perception of neighboring robots and the other due to the communication channels. When defining complexity measures, one thus either has to unify these two types of local interactions, or define two different complexity costs associated with them. Hence, it is natural to ask whether these interactions differ fundamentally from each other. If we can show that there is no fundamental difference, it will simplify our task of characterizing complexity in terms of local interactions by not explicitly mentioning the cause of the interactions. We briefly explore this issue in this section.

Since we are interested in this issue from an information theoretic point of view, we pose the following problem in an information theoretic setting. Let X, Y be two random variables. We will denote by $I(X;Y)$, the amount of information gained about X by knowing Y. The entropy of each random variable will be denoted by $H(X)$ and $H(Y)$ respectively, and $I(X;Y) = H(X) - H(X|Y) = H(Y) - H(Y|X)$ [11], [12]), where $X|Y$ and $Y|X$ are conditional random variables. If a variable Z of M components is defined over a finite field, we will refer to its space as the lattice $\mathbb{Z}_{k_1} \times \mathbb{Z}_{k_2} \cdots \times \mathbb{Z}_{k_M} \subset \mathbb{R}^M$ to emphasize quantization.

Problem 3.1: Suppose the state of a system $X = [x_1, x_2, \ldots x_M]^T \in \mathbb{R}^M$ is measured by sensor $\mathcal{S}$, providing the measurements $Z = [z_1, z_2 \ldots z_M]^T \in \mathbb{Z}_{k_1} \times \mathbb{Z}_{k_2} \cdots \times \mathbb{Z}_{k_M} \subset \mathbb{R}^M$, where $k_i \in \mathbb{N}$ for $1 \leq k \leq M$. Knowledge about X is also transmitted by a remote agent over a communication channel $\mathcal{C}$ as a vector $Y = [y_1, y_2, \ldots y_M]^T \in \mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \cdots \times \mathbb{Z}_{N_M} \subset \mathbb{R}^N$, where $N_i \in \mathbb{N}$ for $1 \leq i \leq N$. Here, the state x_i is assumed to be described by y_i. Each component y_i of Y is encoded independently of other components, and each symbol in each component is equally likely. i.e. $p_i(y_i) = \frac{1}{N_i}$. Then, we would like to ask the following question: Does there always exist a virtual sensor S' which provides the same information as the communication channel $\mathcal{C}$?

The answer to this question is affirmative as show below:

Proposition 3.1: For any communication link $\mathcal{C}$ that satisfies the assumptions in Problem 3.1, there always exists a virtual sensor S' that provides the same information as the communication channel.

Proof: By the setup in Problem 3.1, we have

$$I(X;Y) = \log_2(\prod_{i=1}^{N} N_i).$$

We construct our equivalent "virtual sensor" $\mathcal{S}'$ as follows. Let the virtual sensor give measurements $Z' \in \mathbb{Z}_{N_1} \times \mathbb{Z}_{N_2} \cdots \times \mathbb{Z}_{N_M} \subset \mathbb{R}^M$, with resolutions

$$\begin{aligned} \Delta z'_i &= \frac{\max(y_i) - \min(y_i)}{N_i} \\ f(z'_i - x_i) &= f(x_i | i \Delta z') = \frac{1}{\Delta z_i} \end{aligned}$$

Then it can be directly verified that

$$I(X;Z') = H(X;Y) = \log_2(\prod_{i=1}^{N} N_i).$$

■

However, we would like to show the opposite as well, namely the problem of creating a "virtual" communication channel $\mathcal{C}'$ equivalent to a given sensor. If $I(X;Z)$ is the amount of information gained about X by measurement Z, and there exists a positive integer k such that

$$k = 2^{I(X;Y)} \in \mathbb{Z}^+,$$

then we can build a virtual communication channel $\mathcal{C}'$ using any factorization of k

$$k = k_1.k_2.\ldots.k_K, \qquad k_i \in \mathbb{Z}^+.$$

However, it is usually the case that $I(X;Y)$ is a non-integer due to the choice of real valued continuous, non-constant distributions. Therefore it may not always be possible to construct the virtual channel, using this "trick". But it is clear that the two modes of interaction have no fundamental difference from an information-exchange point of view. Therefore, we assume that we can talk about sensors and communications channels interchangeably. Note that this similarity is information theoretic and not not physical. There are many issues regarding occlusions and multi-hop protocols that must be taken into account to show physical equivalence.

IV. Complexity of Robot Formations

We now consider the problem of defining a complexity measure for multi-agent robot formations. As explained above, it makes sense to relate the complexity measure to the total amount of information flowing in the system. It should further be noted that this information exchange among agents is a dynamic quantity and depends on the distributed algorithm executed by the system.

A multi-agent formation is an evolving structure in both time and space. In space, it is dynamic due to the motion of the robots, which leads to the establishment of new interactions and the termination of old ones. This spatial relationship can be captured by a connectivity graph as explained in Section II. However, the establishment of a local interaction does not mean that this interaction is present for all time. The information exchange at a particular time depends on *protocols* (e.g. [13], [14]), which may make the information interchange not only non-constant, but also non-deterministic. Therefore, it would be appropriate to refer to a quantity describing the time rate of information exchange. We call this quantity, the *information flow*, and refer to the complexity of a formation as the total information flow in the system.

A. Protocols and Information Flows

Suppose $X_j \in \mathbb{R}^N$ is a state associated with an agent j, which agent i wants to acquire by perception or communication. Let $Z_{j,i} \in \mathbb{Z}_{k_1} \times \mathbb{Z}_{k_2} \cdots \times \mathbb{Z}_{k_N} \subset \mathbb{R}^N$ be the measurement of a sensor $\mathcal{S}$ by agent i. Information about X_j is also transmitted by agent j over a communication channel $\mathcal{C}$ as $Y_{j,i} \in \mathbb{Z}_{p_1} \times \mathbb{Z}_{p_2} \cdots \times \mathbb{Z}_{p_N} \subset \mathbb{R}^N$, where $p_i \in \mathbb{N}$ for $1 \leq i \leq N$. If we consider $X_j, Z_{j,i}$ and $Y_{j,i}$ as random processes, then we can define the *information flow*, as the time rate of information exchange taking place at a certain agent, i.e.

$$F_{i,j}(t) = \frac{dI(X_j; Z_{j,i}, Y_{j,i})}{dt}. \qquad (1)$$

There are several technical difficulties associated with the definition in Equation (1). The random processes are always discrete in time, because both the perception and communication process are discrete. In the most general case, the packets arrive (or measurements are taken) according to some *protocol*, which defines the time of arrival. The situation is further complicated by the fact that the information exchange may be completely asynchronous, both among different agents as well as between measurements and communication of the same state for one agent. The actual communication exchange takes place as a burst after possibly long unequal intervals. But, in this paper, we assume that the information flow for a single exchange should be considered as the information gained between two consecutive exchanges, averaged over the time interval.

With these considerations, we assume that if the information flow is well defined according to a particular protocol, then we can define the intrinsic structural complexity of a formation as follows.

Definition 4.1 (Structural Complexity of a Formation): The structural complexity of a formation $\mathcal{F} = (X_1, X_2, \dots X_N) \in \mathcal{C}^N(\mathbb{R}^2)$ is defined as:

$$C(\mathcal{F}) = \sum_j \sum_{i \neq j} F_{i,j}(X_j),$$

where each $F_{i,j}$ is defined according to some given communication protocol.

Since, the presence of protocols implies that every interaction is not active during a certain time period, the intrinsic complexity is bounded above by a quantity that assumes that all interactions are active for all time. This bound is in-fact a complexity associated with a *broadcast protocol*, defined below.

Definition 4.2 (Synchronous Periodic Broadcast Protocol): Suppose each agent j transmits its state $X_j, j \neq i$ to all other agents as Y_j after every Δt seconds. The time Y_j takes to reach agent i is some integer multiple $k_{i,j}$ of Δt, where $k_{i,j}$ is the number of "hops" in the communication. Also, let the measurement $Z_{j,i}$ of remote state X be periodically taken every Δt seconds. Then this protocol of communication among agents is called the *Synchronous Periodic Broadcast Protocol*.

If Δt is the minimum permissible time for information exchange in the system (due to either bandwidth, sensor update interval, or algorithm execution cycle), then we can easily see that protocols of synchronous information exchange that are more selective than the broadcast protocol would result in a decrease of the total information flow. If we denote the complexity of a formation, associated with the broadcast protocol as $C_B(\mathcal{F})$, then

$$C_B(\mathcal{F}) \geq C_P(\mathcal{F}),$$

where $C_P(\mathcal{F})$ is the complexity for some arbitrary protocol. $C_B(\mathcal{F})$ therefore gives the worst case complexity associated with a particular formation. The information flow of a remote state X_j at agent i, according to this protocol, is

$$F_{i,j}(X_j) = \frac{I(X_j; Z_{j,i})}{\Delta t} + \frac{I(X_j; Y_j)}{k_{i,j}\Delta t} \text{ bits/sec}, i \neq j.$$

From the discussion in Section III, it is clear that it is always possible to create a virtual sensor $\mathcal{S}'$ such that $I(X_j; Y_j) = I(X_j; Z_j{}')$. Therefore, we will refer to the information flows with reference to sensors only, and write the information flow as

$$F_{i,j}(X_j) = \frac{I(X_j; \mathcal{Z}_{j,i})}{k_{ij}\Delta t}, \qquad (2)$$

where $\mathcal{Z}_{j,i} = [Z_{j,i}, Z'_j]$, in order to emphasize that we are referring to sensors only.

B. Complexity and Connectivity Graphs

We now study the interesting relationship between the structural complexity defined above and an alternative description of complexity based on connectivity graphs of formations. The first interesting connection can be seen from the definition of the broadcast protocol. The number k_{ij} defined as the number of hops in the communication between agents hints at the network topology between the agents. But, *the connectivity graphs defined in Section II is exactly this network topology.* Furthermore, it may be reasonable to ask if k_{ij} is a unique number for any two agents, since the same information my be exchanged by different hopping paths. This corresponds to different paths in the connectivity graph. Since the information flow in Equation 2 depends on k_{ij}, it must be made clear what path we are using. But, since we are interested in distributed multi-agent algorithms, it cannot be assumed that global information about the network topology (i.e. the connectivity graph of the formation) is available all the time to all agents, so that the hopping paths are unique[2]. Instead, in the broadcast scenario, the information about X_j reaches a remote agent i via all possible hopping paths between them, so that

$$F_{i,j}(X_j) = \sum_{p=1}^{P_{ij}} \frac{I(X_j; \mathcal{Z}_{j,i})}{k_{p,ij}\Delta t},$$

where P_{ij} is the total number of paths, and $k_{p,ij}$ is the length of an individual path, p. If k_{ij} is the smallest path between the agents, i.e. a geodesic in the corresponding connectivity graph, then

$$F_{i,j}(X_j) \leq deg(v_j)\frac{I(X_j; \mathcal{Z}_{j,i})}{k_{ij}\Delta t}.$$

This is the case since even though multiple paths may reach a robot in $star(v_j)$, only one information exchange takes place between that robot and robot j. Furthermore the complexity $C_B(\mathcal{F})$ is bounded above as

$$C_B(\mathcal{F}) \leq \sum_j \sum_{i \neq j} deg(v_j)\frac{I(X_j; \mathcal{Z}_{j,i})}{k_{ij}\Delta t}.$$

We now assume that the states exchanged by all agents are of the same type and encoded in the same way. Therefore $I(X_j; \mathcal{Z}_{i,j}) = \gamma$, i.e. the mutual information is constant for all i, j. Also, note that $k_{ij} = 1$ if v_i, v_j make an edge in the connectivity graph i.e. when agent j can be directly sensed (or communicated with) without an additional hop. We can also write this in standard graph theory notation as $v_j \in star(v_i)$ [15], [16]. Using this notation, we have:

$$C_B(\mathcal{F}) \leq \frac{\gamma}{\Delta t}\sum_i \left(\sum_{\substack{v_j \in \\ star(v_i)}} deg(v_j) + \sum_{\substack{v_j \notin \\ star(v_i)}} \frac{deg(v_j)}{k_{ij}} \right).$$

[2]Network discovery may be possible eventually, but not guaranteed for all time.

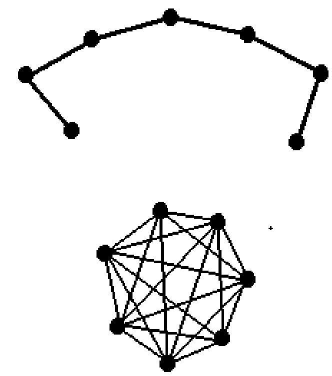

Fig. 1. δ-chain and complete graph for 7 vertices.

It should further be noted that if $v_j \in star(v_i)$, the exact path of communication is *always* known, and the broadcast to other nodes is not necessary. Therefore we can make this bound tighter

$$C_B(\mathcal{F}) \leq \frac{\gamma}{\Delta t}\sum_i \left(deg(v_i) + \sum_{v_j \notin star(v_i)} \frac{deg(v_j)}{k_{ij}} \right),$$

where $\sum_{v_j \in star(v_i)} 1 = deg(v_i)$. Compare this to the complexity defined on a graph G, in the context of molecular chemistry [4], given as

$$C(G) = \sum_{v_i \in \mathcal{V}} \left(deg(v_i) + \sum_{v_j \in V, v_i \neq v_j} \frac{deg(v_j)}{d(v_i, v_j)} \right),$$

where $d : \mathcal{V} \times \mathcal{V} \rightarrow \mathbb{R}^+$ is some distance function defined between vertices. Therefore, we get

$$C_B(\mathcal{F}) \leq \frac{\gamma}{\Delta t} C(\Phi_N(\mathcal{F})),$$

where $\Phi_N(\mathcal{F})$ is the connectivity graph of the formation. This relationship leads to the following interesting observation: *The complexity of the connectivity graph of a formation is a (tight) upper bound for the worst case complexity associated with an arbitrary protocol of communication in a multi-agent formation*. Therefore the study of the structural complexity of robot formations is closely related to the complexity of their connectivity graphs.

C. Simple and Complex Connectivity Graphs

The complexity measure on connectivity graphs gives a good comparison between different formations. While it is difficult to produce an absolute order on all possible connectivity graphs, it distinguishes simple graphs from the more complex. We will prove below that the complete graph is the most complex connectivity graph for a fixed set of vertices, whereas a δ-chain [5], which is the line graph (i.e. a Hamiltonian path on all vertices), is the least complex connected connectivity graph. (See Fig 1.)

The conclusion that the complete graph is the most complex graph is not surprising and conforms to our intuition, as it has the maximum number of local interactions between any set of vertices. The characterization of the most simple graph is however an interesting result and gives the justification of the δ-chaining algorithms that we have developed as a benchmark problem in our study of distributed algorithms [5], [6], [7].

Consider a connectivity graph $G_N = (\mathcal{V}, \mathcal{E})$ on N vertices, with the complexity measure

$$C(G_N) = \sum_{v_i \in \mathcal{V}} \left(deg(v_i) + \sum_{v_j \notin star(v_i)} \frac{deg(v_j)}{k_{ij}} \right).$$

If we add another vertex v_{N+1} to G_N, we get a graph on $N+1$ vertices G_{N+1}. We can also form new edges between v_{N+1} and vertices in V so that the complexity of the new graph is perturbed as

$$\begin{aligned} C(G_{N+1}) = &\sum_{v_i \in \mathcal{V}} (deg(v_i) + \Delta deg(v_i) \\ &+ \sum_{\substack{v_j \notin star(v_i) \\ v_j \in \mathcal{V}}} \frac{deg(v_j) + \Delta deg(v_j)}{k_{ij} + \Delta k_{ij}} + \frac{deg(v_{N+1})}{k_{i,N+1}}) \\ &+ deg(v_{N+1}) + \sum_{v_m \notin star(v_{N+1})} \frac{deg(v_m) + \Delta deg(v_m)}{k_{mj} + \Delta k_{mj}}, \end{aligned}$$

where $\Delta deg(v_i)$ is the change of degree at vertex v_i caused by the addition of a new vertex, and Δk_{mj} is the corresponding decrease in the shortest path between vertexes v_m and v_j.

It can be seen that adding a vertex always *increases* the complexity of the graph, as all perturbations are additive. It is therefore straightforward to capture the minimum or maximum perturbation that can be done by adding a vertex.

Theorem 4.1: If G is a connected connectivity graph then the complexity of G is bounded above and below as

$$C(\delta_N) \leq C(G) \leq C(\mathcal{K}_N),$$

where δ_N is the δ-chain on N vertices, and $\mathcal{K}_N$ is the complete graph.

Proof: We prove the theorem by induction. Suppose it is true that $C(G) \leq C(\mathcal{K}_N)$ for $G \in \mathcal{G}_{N,\delta}$. Note that for any vertex v_i in the graph, $deg(v_i) \leq N$. For $\mathcal{K}_N$, $deg(v_i) = N$ for all vertices. Therefore the maximum number by which any degree can be perturbed in $\mathcal{K}_N$ is 1. The perturbation will be maximized if all degrees are perturbed by 1. Similarly, in $\mathcal{K}_N$, $k_{ij} = 1$ for all pairs of vertices. The maximum perturbation will take place when the relation $k_{ij} = 1$ still holds for all pairs after addition of new vertex, i.e. all vertices are directly connected. It can be easily seen that this can only be accomplished by adding edges between all vertices in $\mathcal{K}_N$ and the new vertex to make the graph $\mathcal{K}_{N+1}$. This proves that $C(G) \leq C(\mathcal{K}_N)$ for all N.

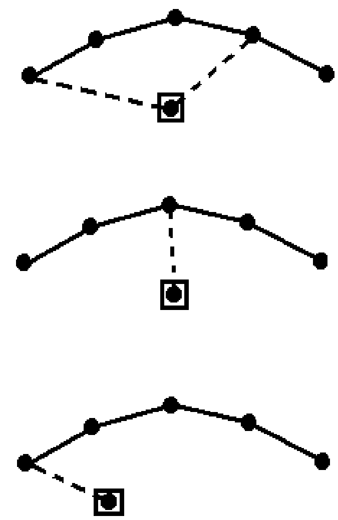

Fig. 2. Different ways to add a new vertex to δ_5

We repeat the induction argument for the lower bound as well. Suppose it is true that $C(\delta_N) \leq C(G)$ and we look at the perturbation equation of δ_N for minimum increase. (See fig 2.) Since all terms in the perturbation equation are non-decreasing, it would be least perturbed, if each individual term is minimally increased. In order to produce a connected graph, $deg(v_{N+1}) \geq 1$. (If connectedness was not required, we would have added another vertex with 0 degree). For minimum increase, set $deg(v_{N+1}) = 1$. This would also mean that $\Delta deg(v_i) = 0$ for all v_i in δ_N except one. This corresponds to addition of exactly one edge to the old graph, δ_N. However this edge can be added to any of the N vertices. Note that this edge addition may disturb the shortest paths k_{ij} between node pairs v_i, v_j. (The paths cannot be lengthened by edge addition). If that happens, terms of the form $deg(v)/k$ will get bigger. The only way to avoid this is to add the edges to either end of the chain. Therefore $\Delta k_{ij} = 0$ for all $1 \leq i, j \leq N$. This also maximizes $k_{i,N+1}$ for all $1 \leq i \leq N$ so that $deg(v_{N+1})/k_{i,N+1} = 1/k_{i,N+1}$ are minimized for all $i \leq N$. This shows that if the edge is added to a vertex which is not an end point, it results in an addition of degrees as well as a decrease in k_{ij} for some vertices, again resulting in increase of complexity. Therefore, the optimal way to add the edge is to add the edge at its ends, which results in another delta chain δ_{N+1}. ■

The consequence of this theorem is that the δ-chain is the simplest formation that can be formed over a fixed number of agents. This perhaps explains why humans like to make queues and birds fly in V-formations, both of which are essentially δ-chains and require minimum coordination among individuals. We will use this result in the future to justify various δ-chaining algorithms that are part of our current investigations of connectivity graphs.

V. Conclusions

In this paper, we have presented a complexity measure for studying the structural complexity of robot formations. We have based this complexity measure on the number of local interactions in the system due to perception and communication. We showed that from an information theoretic point of view, perception and communication are fundamentally the same and should therefore not be discriminated when defining local interactions. We also showed that the broadcast protocol corresponds to the worst case complexity for a given formation and serves as an upper bound. We further noted that this upper bound is remarkably similar to the complexity measure of graphs defined in the context of molecular chemistry. This complexity measure on graphs was further explored to characterize the most complex and most simple graphs for a fixed number of vertices. We found that the complete graph and the δ-chain are the extremal complexity graphs.

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